Journal of Intelligent Material Systems and Structures

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Journal of Intelligent Material Systems and Structures 2008 19: 845 originally published online 17 October 2007
DOI: 10.1177/1045389X07082024

Model Predictive Control of a Two Stage Actuation System using Piezoelectric Actuators for Controllable Industrial and Automotive Brakes and Clutches

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ABSTRACT: High bandwidth actuation systems that are capable of simultaneously producing relatively large forces and displacements are required for use in automobiles and other industrial applications. Conventional hydraulic actuation mechanisms used in automotive brakes and clutches are complex, inefficient and have poor control robustness. These lead to reduced fuel economy, controllability issues and other disadvantages. Recently, a two-stage hybrid actuation mechanism was proposed by combining classical electromechanical actuators like DC motors and advanced smart material devices like piezoelectric actuators. This article discusses the development and implementation of a model predictive control methodology for controlling this two-stage actuation system in tracking various reference inputs. Additionally, this methodology also employs a unit-step delayed disturbance estimate to account for actuator hysteresis, other nonlinearities and unmodeled dynamics in the system. Finally, the article highlights the effectiveness of this control methodology experimentally by tracking various reference inputs.

Key Words: brakes, clutches, smart materials, piezoelectric actuators, model predictive control, two-stage actuation.

INTRODUCTION

CURRENT state-of-the-art automotive systems rely extensively on controllable actuation mechanisms (primarily hydraulic) to achieve good braking and clutching performance. The different actuation systems for brakes and clutches currently utilized in industry vary from the classic hydraulic mechanisms (Hai-Fraj and Pfeiffer, 1999; Gmbh, 2000; Juvinall and Marshek, 2000; Wang et al., 2001; Morselli et al., 2003) to the latest Magnetorheological (MR) fluid clutches (Neelakantan and Washington, 2002; Kavlicoglu et al., 2002). Other controllable clutch/brake actuation mechanisms include those using magnetic particles, pneumatics, electromagnetism, magnetic hysteresis, Eddy current and Electrorheological fluids to name a few (Vorobyeva, 1965; Patras and Russel, 1992; Mikhaeil-Boules, 1994). However, all of these mechanisms have their own advantages and disadvantages. For instance, hydraulically actuated clutch control systems, which are widely used in automobiles equipped with automatic transmissions, have an advantage in high power density but they also have disadvantages like poor efficiency, low robustness due to the varying bulk modulus of pressurized fluid, the requirement for pumping hardware, and a complicated system design due to intricate fluid passages containing valves with moving parts (Haj-Fraj and Pfeiffer, 1999). Similarly the MR fluid clutches, though simpler compared to the hydraulically actuated clutches, have uncertain operation characteristics at high rotational speeds due to the effect of centrifuging of the micron-sized magnetizable particles (Neelakantan and Washinton, 2002). Also electromagnetically actuated clutches require large control power due to the presence of an air gap between the rotor and the armature. Likewise, magnetic hysteresis and Eddy current clutches have very low torque-to-size ratio characteristics. Hence, smart material actuators like piezoelectric actuators with high force capabilities and quick response characteristics may provide an effective solution to the controllable clutching and braking needs in many industrial and commercial applications.

An automotive friction clutch application works by the compression action of two sets of friction discs, one attached to the input shaft and the other attached to the output shaft. For such a typical friction clutch/brake, the required stroke is approximately 2 to 3 mm and the required force is approximately a few kiloNewtons,

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Figures 1–3 and 5–16 appear in color online: http://jim.sagepub.com

1045-389X/08/07 0845–13 $10.00/0 DOI: 10.1177/1045389X07082024

ranging from about 4 kNs for a typical clutch engagement to at least 10 kNs for a brake engagement (Gmbh, 2000; Juvinall and Marshek, 2000; Wang et al., 2001; Morselli et al., 2003). Recent developments in the field of smart materials like Piezoelectric and Magnetostrictive materials have led to the development of advanced actuators with low to moderate stroke and high force capabilities. The best piezoelectric actuators that are commercially available are able to produce very high forces of the order of 10 to 80 kN. Their maximum stroke, however, is limited to around 100–300 μm (Su et al., 2000; See also web page www.americanpiezo.com and www.physikinstrumente.com). Previous work involving smart material actuators for clutching/braking applications involves the use of levers and hydraulic amplification mechanisms for stroke amplification of the actuators (Thorney et al., 1991; Gogola and Goldfarb, 1999). These methods, however, lead to difficulties like high mechanical tolerance requirements, reduced efficiency, lower fatigue life and a significant equivalent reduction in force capabilities. However, one may utilize the dual stage characteristics of the force stroke relation of a typical clutch/brake actuation process to develop a novel actuation mechanism. This idea is discussed in detail in the section entitled ''Two-stage hybrid actuation system''. Analogous ideas of dual stage actuation have been used in (i) computer disk hard drives to improve bandwidth of the read/write process (Lee et al., 2000) and (ii) noncircular cam turning processes (Kim et. al., 2001). For the hard disk drives, a voice coil motor is used in conjunction with a piezoelectrically actuated read/write head to maintain the overall range of motion of the device and simultaneously improve overall bandwidth of the system.

Using smart material actuators in automotive applications require actuator models that describe the actuator properties and are conducive to control system design. Most models, however, fall far short of these requirements. Many analytical and phenomenological models like the homogenized free energy model (Smith et al., 2003) are excellent in capturing the performance of the actuator accurately but are too complicated and nonlinear for real time linear control system design. Similarly a simple linear model may allow for easier control system design but the accuracy of this model may fall short in providing the required system performance (Giurgiutiu and Pomirleanu, 2001). One of the most popular nonlinear techniques is the general phenomenological method, because it is accurate, easy to implement and can be applied to a broad class of nonlinear actuators. Ge et al. have demonstrated their use in control design by using phenomenological modeling techniques to design a simple PID controller (Ge and Jouanch, 1996). Though these techniques work well for a specific actuator, they fail to be robust when applied to different actuators with similar characteristics. Additionally, these models require significantly large amounts of experimental data to be stored in computer memory. An approximate model using a bounded delay and gain has been proposed by Tsai et al. (2003). This model is attractive to researchers because it is based on the actual reason for hysteresis in piezoelectric actuators, which is inherent delays due to domain wall shifting. This model has also been used by Neelekantan et al. in designing and implementing a robust controller using the Internal Model Control principle (Neelakantan et al., 2005). While this controller shows good results, the system tends to go unstable when the speed of response is increased. This method also lumps the time-varying delay and gain in the system into a linear time invariant system with uncertainties. This assumption is valid for the time-varying delay as proven by Lincoln et al. (2004) but is not valid for the time varying gain. Also this model does not accurately account for the nonlinearities in the system including higher order reversal curves in the hysteresis properties of the actuator.

Piezoelectric actuator based systems can be fundamentally modeled as a linear system with time-varying but unknown uncertainties. The time-varying uncertainties that depend on the control input can virtually account for the hysteresis nonlinearities in the system. Hence the controller to be designed must provide a framework for handling time-varying uncertainties. Also since the uncertainties depend on the control action some means of optimal control action is desired that incorporates weighted control actions. This can virtually ensure system stability in the presence on control-dependent-uncertainties. Since Model Predictive Control (MPC) is a control strategy that is formulated to produce optimal or sub-optimal control actions using discrete-time models, it also provides a framework to instantaneously estimate the value of the time-varying disturbances and uncertainties in real-time to be used in the computation of the control action. Thus the distinct benefit of MPC over other control methods for systems with actuator nonlinearities is its ability to produce optimal control actions using a receding window horizon prediction methodology that can account for time-varying disturbances and uncertainties in real-time.

The research in this article adapts and implements the Model Predictive Control or MPC methodology (Rossiter, 2003). This method has become quite popular among process control engineers and used in most of the chemical process industry (Rossiter, 2003). As the name suggests, this controller methodology uses a discrete-time model of the dynamic system to predict the system performance in a predefined length of time in the future in order to arrive at the optimal control action at each instant of time. However, though this method has an implicitly built-in robustness property, it still requires a relatively accurate model to provide premium performance especially in the case of reference tracking. This article discusses in detail the new two-stage actuation mechanism with specific attention to the adaptation and implementation of the MPC methodology. The time constants of chemical process systems are usually on the order of a few seconds to a few minutes, with a low sampling frequency, giving the controller plenty of time to compute the computationally intensive optimal control actions at each time step (Pike et al., 1996; Morari, 1997; Rossiter, 2003); nevertheless, the dual stage actuation mechanism in this study is relatively much faster with a required settling time lower than 0.2 s and a sampling time of 1 ms or smaller. Hence this puts a huge bottleneck on the complexity of the optimization problem that the controller can perform at each time step. This means that the system equations must be formulated in a manner so that the optimization problem has a closed form solution that may be quickly applied at each time step and must also virtually account for the nonlinearities in the system like hysteresis. The control system is implemented using a unit-step delayed disturbance estimate that has been successfully used in discrete-time sliding mode control (Su et al., 2000; Jalili-Kharaajoo and Fazaie, 2003; Jalili-Kharaajoo, 2004). Finally, experimental results are shown in section 4 illustrating the effectiveness of this methodology in controlling piezoelectric actuator systems.

TWO-STAGE HYBRID ACTUATION SYSTEM

Operating Principle of the New Two-Stage Actuation Mechanism

The force-stroke requirement of a typical clutch/brake actuation process is shown below in Figure 1. It is apparent that the force/stroke requirement may be split into two regions. In region I, the required stroke is high but the force required to produce the stroke is relatively low. Physically, this region represents the movement of the brake pad or the input clutch pack close enough to the rotating disc or the output clutch pack, respectively. In other words, the major component of the stroke is used to move the clutch pack or the brake pad to the vicinity of the rotating component. While the stroke required for this movement is relatively large, the force required to perform this action is comparatively small. In region II, the required force increases to a very high value within a small increment in the stroke. This region represents the period when the brake pad or the input clutch pack is compressed against the rotor or the output clutch pack respectively to produce the necessary braking or clutching action.

One can see that the entire actuation process is split into two phases with contrasting force-stroke requirements. Hence two different actuation devices

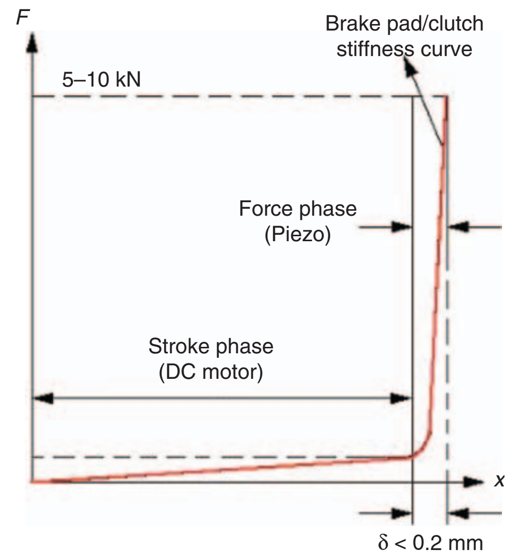

***Figure 1.** Typical force–stroke relation in clutching/braking action.*

may be utilized to satisfy the requirements in the two phases. For instance, in the first phase, classical DC motors may be employed to provide the required large stroke with relatively low force. In the second phase, piezoelectric actuators may be used to produce the relatively higher force with the relative smaller stroke, thereby completing the actuation process. Two different actuators can be coupled together using a mechanical coupling device either in a series or a parallel mode. In the series mode both actuators will bear the same load and in the parallel mode, both the actuators will produce the same displacement. However, by using the appropriate coupling mechanism (e.g., a lead screw-nut or a worm gear) between the motor and piezoactuator, the hybrid system can produce the required force-displacement characteristic. Both of these coupling mechanisms possess a *self-locking* property. By virtue of this property, the input and output members of the coupling cannot be reversed. For instance, in the lead screw-nut assembly, the lead screw can drive the nut but the system will lock when the nut tries to drive the lead screw (Neelakantan and Bucknor, 2004).

Design and Modeling

The objective of modeling the system is to arrive at a relatively simple model that captures the important characteristics of the system while also providing a framework for easier robust controller design. Hence, the model development is centered on the construction of equations that lend themselves to controller development. The system model consists of the integration of the individual models of the following components of the mechanism: the DC motor, lead screw and nut assembly, piezoactuator, load (spring element or brake pad), and other coupling elements as shown in Figure 2(a) and (2b).

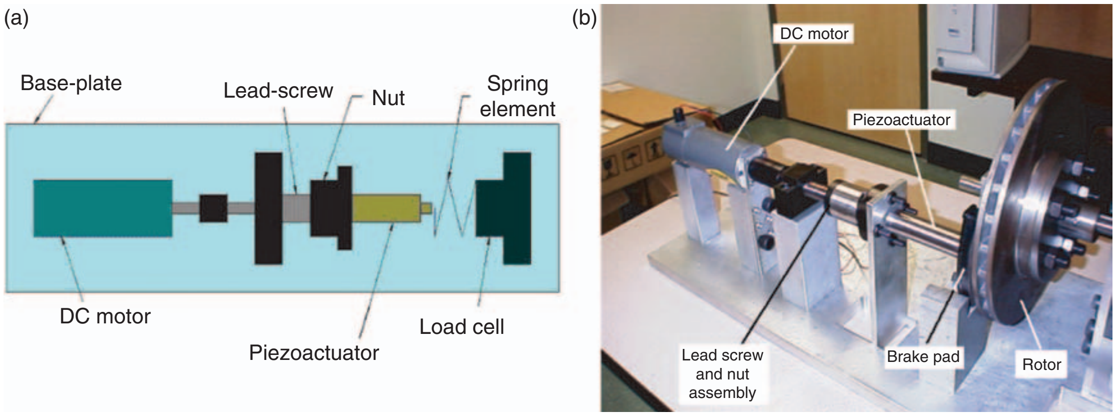

Figure 2 (a, b). *DC Motor and piezoactuator based two-stage high force/high stroke actuator.*

The modeling is again divided into two parts namely the stroke phase and the force phase. The task of the stroke phase is to bring the clutch plate or the brake pad close to the rotating end of the device. In this phase, the DC motor serves as the actuator. For a given design the nominal final position to which the motor must be driven is known approximately, but the actual location of the final point may vary depending on factors like wear and tear on the device elements and the stiffness property of the load. The transition from the stroke phase to the force phase is critical because of the extremely small displacements of the piezoelectric actuators. If the transition is made too early, before the brake pads/clutch plates are brought into contact with each other, the force phase becomes ineffective because the stroke of the piezoelectric actuators will be exhausted in achieving the additional displacement needed to reach the actual transition point. If the transition point is detected too late, though the force phase functionality requirements can be achieved, it may lead to damaging the DC motor by extended stalling.

DC MOTOR AND LEAD SCREW AND NUT ASSEMBLY

The DC motor model is well known and is governed by two simple equations, one for the motor circuit and the other for the torque balance. The DC motor equations are

$$J_{\text{total}}\ddot{\theta} + f_r\dot{\theta} + T_L = Ki \tag{1}$$

$$L_m\frac{\mathrm{d}i}{\mathrm{d}t} + Ri + K\dot{\theta} = U \tag{2}$$

where J_{total} = the total inertial load on the motor, R = motor armature resistance, L_m = motor armature inductance, K = motor constant, f_r = damping friction coefficient, i = motor current, U = motor control voltage. Though the system states include both the angular velocity '$\dot{\theta}$' and current 'i', only the current is measured. The term T_L is the additional load-torque term included to model stalling of the motor at the transition point. The controllability and the observability matrix show that the above system is completely controllable and observable which means that complete information about the system states may be found by measuring only the current. The operating strategy of the first phase of the actuation process using the DC motor is to apply a constant voltage to the DC motor until the transition point is reached. The transition point occurs when the motor is stalled as the lead screw assembly carrying the piezoelectric actuator and the brake pad contacts the rotating unit. The current drawn by the motor during this transition increases rapidly due to stalling. The current spike is sensed and a controller instantaneously stops the voltage supply to the DC motor when the current exceeds a predefined current limit. This provides a preload on the brake pad compressed against the rotating member. This preload value, which increases the effectiveness of the piezoelectric actuator, is determined based on predefined current and voltage limits.

PIEZOELECTRIC ACTUATOR MODEL

This section concerns the design and modeling of the second phase of the actuation process and is the most critical since the piezoelectric actuator produces most of the required force in the actuation process. Once the transition point is detected, the force phase is executed using piezoelectric actuators. The force exerted by the piezoelectric actuator compresses the clutch-pack or the brake pad. The final operating/settling actuator displacement of the force phase is determined by the intersection of the force-displacement curves of the piezoelectric actuator and the clutch-pack/brake pad stiffness. The piezoelectric actuator produces a very high force when the actuator is blocked from producing any stroke. For a given voltage command, the force capability of the actuator decreases to zero linearly as the stroke is allowed to increase to its maximum value. Figure 3 illustrates the properties of a piezoelectric actuator acting against a spring load. As shown, the force-displacement output of the piezoelectric actuator increases approximately linearly with the

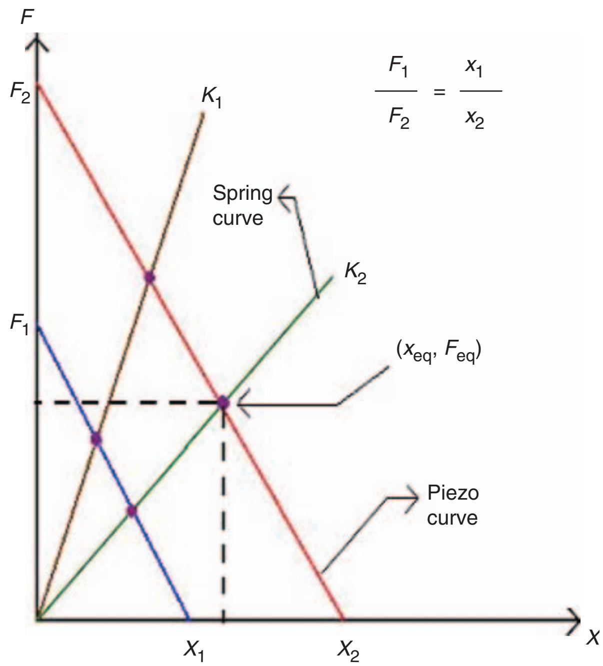

Figure 3. *Force–Displacement characteristic of piezoelectric actuator loaded against a spring.*

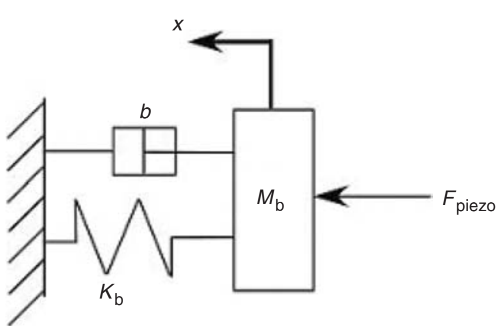

Figure 4. *Simple model for Force Phase dynamics.*

applied voltage. Hysteresis in piezoelectric actuators is estimated to be approximately 10–15% and will be compensated for by closed-loop control. The expressions for the equilibrium operating point location are

$$F_{eq} = F_{bl}\left(\frac{K_{\text{spring}}}{(F_{bl}/x_{\max}) + K_{\text{spring}}}\right),\ x_{eq} = \frac{F_{eq}}{K_{\text{spring}}}. \quad (3)$$

The clutch-pack dynamics during the force phase can be modeled using a mass-spring-damper-actuator system. The friction in the system may be assumed to be a small value and can be determined through experiments. Figure 4 shows a simple model of the force phase. Based on this model, the equation of motion for the system may be deduced as follows. Using Figure 4 and equations (5–7), M_b, K_b, b, and x represent the mass, clutch-pack stiffness, friction/damping in the system and the displacement of the clutch-pack, respectively. Starting from the constitutive relations for Piezoelectric actuators, we have

$$T_i = c_{ij}^E S_j - d_{ki} c_{ji}^E E_k \quad (4)$$

where T represents the stress, S represents the strain in the piezo and E represents the applied Electric field, c represents the material modulus at constant E and d is the Piezoelectric constant. In this case, all the stresses and strain may be assumed to be acting only in the three direction and the applied stress is from the spring and the damping in the system. Here, T must equal the total dynamical stress from the system including the mass, stiffness and damping. Hence one may extend the above formulation and write the following dynamical equation describing the system.

$$M_b\ddot{x} + b\dot{x} + K_b x_b = F_{\text{piezo}} \quad (5)$$

$$F_{\text{piezo}} = F_{bl}\left(1 - \frac{x_b}{x_{\max}}\right) \quad (6)$$

$$\Rightarrow M_b\ddot{x} + b\dot{x} + \left(K_b + \frac{F_{bl}}{x_{\max}}\right)x = F_{bl} \quad (7)$$

The blocked force 'F_{bl}' and the free displacement '$x_{\max}$' of the piezoelectric actuator are functions of the applied voltage and their ratio is the voltage-based apparent stiffness (K_p) of the actuator. Assuming linear relationships with the actuator acting in (33) mode, we have, $F_{bl} = (Ad_{33}c_{33}^E/t)U = \beta U$ and $K_p = Ac_{33}^E/nt$, where A is the surface area of the piezoelectric stack actuator, d_{33} is the piezoelectric constant, c_{33} is the material modulus of the actuator, n is the number of layers in the stack actuator and t is the thickness of each layer in the actuator. As is evident, the relation between the applied voltage (U) and the blocked force of the piezoelectric actuator is of utmost importance here. Though linear relations between the applied voltage and blocked force are shown above, these are not completely accurate due to hysteretic and anhysteretic nonlinearities of piezo-actuators. A numerical phenomenological model popularly known as the "Preisach model" has been widely proposed and used successfully by many researchers (Ge and Jouaneh, 1995). The Preisach model utilizes a summation of weighted relay operators to describe the relationship between the actuator's input voltage and its output free displacement or blocked force. This model is very accurate in predicting the response of a specific piezoelectric actuator but it requires a significant amount of experimental data to be stored in computer memory. Though it has been successfully used in tracking control problems using PID (Ge and Jouaneh, 1996), this model makes it difficult to design and implement advanced control strategies due to its numerical structure.

CONTROL METHODOLOGY DEVELOPMENT

An appropriate controller is to be designed for the system so that the system robustly tracks a time-varying

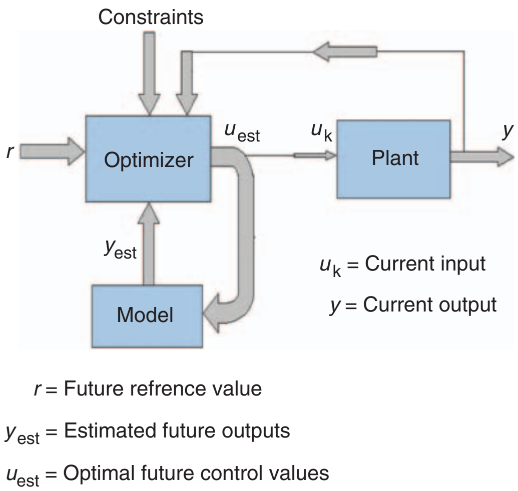

Figure 5. *Schematic of model predictive controller.*

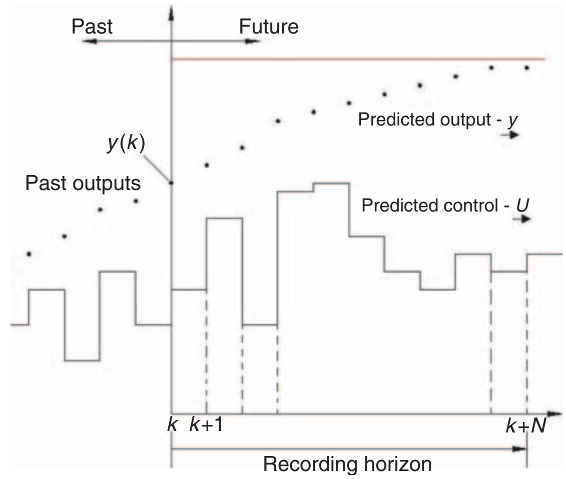

Figure 6. *Illustration of the working of MPC.*

reference force signal. A settling time of less than 0.2 s for a step reference is desirable with offset-free steady state output. Also most of the existing actuators have a tracking bandwidth requirement of up to 5–10 Hz. Hence the dual-stage actuator must also provide satisfactory tracking performance up to 10 Hz. Since the system involves constraints and an accurate hysteresis model that is practical for real-time control applications is not readily available, it is imperative that a real-time controller be designed and implemented for consistent performance. Model Predictive Control, also known as receding horizon control, is an online optimal feedback control methodology used to control linear and nonlinear systems with or without constraints. The advantages and merits of using MPC include the ability to handle constraints and to produce optimal solutions at each control move. The idea here is to develop a control algorithm that uses a receding horizon or window to predict the behavior of the system over a predefined length in the future and to adjust the system control action to produce an optimal solution with respect to a predefined cost function. MPC requires a representative discrete-time model of the process usually in state space form or transfer function form. The working procedure of MPC is to use a discrete time model for the dynamic system that allows a receding window prediction of future outputs based on the current value of the output and future value of inputs. This procedure is then used to arrive at an optimal value of future control inputs relative to a predefined cost criterion. Figures 5 and 6 highlight the concept of the MPC algorithm.

Model Predictive Cotrol is known for its ability to handle uncertainties, constraints and external and internal disturbances to provide excellent controller performance. These properties have been utilized in many process control problems to effectively control dynamic plants in the chemical process industry. However, an accurate model is usually required to implement MPC successfully. Moreover, the time constants of chemical process systems are usually on the order of a few seconds to a few minutes, with a low sampling frequency, giving the controller plenty of time to compute the computationally intensive optimal control actions at each time step (Pike et al., 1996; Morari and Lee, 1997; Rossiter, 2003). However, the dual stage actuation mechanism in this study is relatively much faster with a required settling time lower than 0.2 s and a sampling rate of 1 ms or lower. Hence, this puts a huge bottleneck on the complexity of the optimization problem that the controller can perform at each time step. This means that the system equations must be formulated in a manner so that the optimization problem has a closed form solution that may be quickly applied at each time step and must also virtually account for the nonlinearities in the system like hysteresis. The following analysis details a method to adapt the MPC concept to the dual-stage system in an efficient way.

Model Predictive Control

From Equation (7), the nominal equation of motion for the force phase can be reproduced in state space form as

$$\dot{X} = A_c X + B_c U + d(t) \tag{8}$$

where the state $X = [f \quad \dot{f}]^T$ is the vector of the generated force and its derivative, and

$$A_c = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}, \quad B_c = [0 \quad \beta]^T,$$

with U being the control voltage. Note that an additional term $d(t)$ is added to account for unmodeled dynamics and nonlinearities, including hysteresis, in the system. It is also assumed that this disturbance and its derivative are bounded. Since the hysteretic effects cause the displacement to vary about 15% from the nominal, this technique includes a feature that can account for the effect hysteresis directly. The controller is designed so that the total force from the system, which is the sum of the force produced in the stroke phase and the force phase, tracks a time-varying reference force requirement. At each time instant, it is assumed that the force produced is equal to the product of the brake pad stiffness and the system displacement (X). The following procedure describes the methodology for implementing MPC.

Consider a continuous time dynamic system with disturbance as shown in Equation (8), where "X" $\in \Re^n$ is the state vector, "U" $\in \Re^m$ is the control, "$d(t)$" $\in \Re^n$is the time-varying disturbance vector. A_c and B_c are the system and control matrices of appropriate dimensions. The disturbance is added to model the uncertainties and unmodeled dynamics in the system. Given a sampling time "T", one may convert this continuous-time system given by Equation (8) into an equivalent discrete time system using a standard discretization procedure (Ogata, 1987). This yields

$$X_{k+1} = AX_k + BU_k + d_k \tag{9}$$

$$Y_k = CX_k \tag{10}$$

where

$$A = e^{A_c T},\ B = \int_0^T e^{A_c\tau} B \mathrm{d}\tau \text{ and}$$

$$d_k = \int_0^T e^{A_c\tau} d(kT + T - \tau)\mathrm{d}\tau$$

and, with B and d_k being $O(T)$, meaning it is of the order of T, if d(t) is bounded and Y defined as the measured output. Defining a new state that augments the control input we have

$$Z_k = \begin{bmatrix} X_k \\ U_{k-1} \end{bmatrix}. \tag{11}$$

Now the new state equation is

$$\begin{bmatrix} X_{k+1} \\ U_k \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}\begin{bmatrix} X_k \\ U_{k-1} \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix}\Delta U_k + \begin{pmatrix} I \\ 0 \end{pmatrix} d_k. \tag{12}$$

In other words, this can be written as

$$Z_{k+1=} = \hat{A}Z_k + \hat{B}\Delta U_k + \hat{D}d_k \tag{13}$$

with

$$Z_k = \begin{bmatrix} X_k \\ U_{k-1} \end{bmatrix} \in \Re^{n+m},\ \hat{A} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \in \Re^{(n+m)\times(n+m)},$$

$$\hat{D} = \begin{pmatrix} I \\ 0 \end{pmatrix} \in \Re^{(n+m)\times(n)}$$

and $\hat{B} = \begin{bmatrix} B \\ I \end{bmatrix} \in \Re^{(n+m)\times(m)}$.

The new output equation is given by

$$Y_k = \begin{bmatrix} C & 0 \end{bmatrix}\begin{bmatrix} X_k \\ U_{k-1} \end{bmatrix} = \hat{C}Z_k \tag{14}$$

Extending Equation (14) by one and two steps ahead in time gives

$$Y_{k+1} = \hat{C}Z_{k+1} = \left(\hat{C}\hat{A}\right)Z_k + \left(\hat{C}\hat{B}\right)\Delta U_k + \left(\hat{C}\hat{D}\right)d_k \tag{15}$$

and

$$Y_{k+2} = \hat{C}Z_{k+2} = \hat{C}\hat{A}Z_{k+1} + \hat{C}\hat{B}\Delta U_{k+1} + \hat{C}\hat{D}d_{k+1} \tag{16}$$

$$\Rightarrow Y_{k+2} = \left(\hat{C}\hat{A}^2\right)Z_k + \left(\hat{C}\hat{A}\hat{B}\right)\Delta U_k + \hat{C}\hat{B}\Delta U_{k+1} + \left(\hat{C}\hat{A}\hat{D}\right)d_k + \hat{C}\hat{D}d_{k+1}. \tag{17}$$

Extending this prediction up to N steps ahead in time gives the following matrix relationship

$$\underset{\rightarrow}{Y} = \underbrace{\begin{bmatrix} \hat{C}\hat{A} \\ \hat{C}\hat{A}^2 \\ \vdots \\ \hat{C}\hat{A}^N \end{bmatrix}}_{P} Z_k + \underbrace{\begin{bmatrix} \hat{C}\hat{B} & 0 & \cdots & 0 \\ \hat{C}\hat{A}\hat{B} & \hat{C}\hat{B} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{C}\hat{A}^{(N-1)}\hat{B} & \hat{C}\hat{A}^{(N-2)}\hat{B} & \cdots & \hat{C}\hat{B} \end{bmatrix}}_{H} \underset{\rightarrow}{\Delta U} + \underbrace{\begin{bmatrix} \hat{C}\hat{D} & 0 & \cdots & 0 \\ \hat{C}\hat{A}\hat{D} & \hat{C}\hat{D} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{C}\hat{A}^{(N-1)}\hat{D} & \hat{C}\hat{A}^{(N-2)}\hat{D} & \cdots & \hat{C}\hat{D} \end{bmatrix}}_{L} \underset{\rightarrow}{d} \tag{18}$$

where $\underset{\rightarrow}{Y} = (\,Y_{k+1} \quad Y_{k+2} \quad \cdots \quad Y_{k+N}\,)^T$, $\Delta \underset{\rightarrow}{U} = (\,\Delta U_k \quad \Delta U_{k+1} \cdots \quad \Delta U_{k+N-1}\,)^T$ and $\vec{d} = (\,d_k \quad d_{k+1} \quad \cdots \quad d_{k+N-1}\,)^T$. It is noted that the matrices H and L in Equation (18) have a Toeplitz structure. We thus have used the state space model of the dynamic system to provide a framework for prediction of the system output up to N steps in the future. The controller to be designed is required track a reference input R. Model Predictive Control uses the prediction Equation (18) to define and solve an optimization problem that minimizes an appropriate cost function over the next N steps. Let's define a positive-definite cost function

$$J = \left\| \underset{\rightarrow}{R} - \underset{\rightarrow}{Y} \right\|^2 + \lambda \left\| \underset{\rightarrow}{\Delta U} \right\|^2 \tag{19}$$

where $\underset{\rightarrow}{R}$ represents the future values of the reference inputs up to N steps ahead in time that are to be tracked by the system output. It is noted that the cost function puts a weighting on the change in control effort relative to each step rather than the control effort itself. This is justified for piezoelectric actuator systems since the power consumed by the actuator is approximately proportional to the rate of change of the voltage. In addition, the steady state value of the terms in the cost function must tend to zero. This happens only when the change in control effort is used instead of the actual control input. Substituting Equation (18) into (19) gives,

$$\Rightarrow J = \left\| \underset{\rightarrow}{R} - PZ_k - H\Delta\underset{\rightarrow}{U} - L\underset{\rightarrow}{d} \right\|^2 + \lambda \left\| \Delta\underset{\rightarrow}{U} \right\|^2$$

where the matrices P, H, and L are shown in Equation (18). In order to perform the optimization, we use the condition, $\partial J/\partial \Delta\underset{\rightarrow}{U} = 0$. This yields

$$\left(H^T H + \lambda I\right)\Delta\underset{\rightarrow}{U} = \left[H^T\underset{\rightarrow}{R} - H^T PZ_k - H^T L\underset{\rightarrow}{d}\right]. \quad (20)$$

Hence, the optimal MPC-based controller is given by

$$\Delta\underset{\rightarrow}{U} = \left(H^T H + \lambda I\right)^{-1}\left[H^T\underset{\rightarrow}{R} - H^T PZ_k - H^T L\underset{\rightarrow}{d}\right]. \quad (21)$$

The above equation gives the optimal value of the control changes up to "N" steps ahead in time including the change to be applied at the current instant based on the dynamic system model. The MPC concept, however, requires that we only apply the first value in the list of "N" controller values given by Equation (21). Once the system responds to this control action, the same procedure is performed at the next sampling instant and so on. The value of the control effort at the instant k is given by

$$U_k = U_{k-1} + \Delta U_k \quad (22)$$

where $\Delta U_k = [I \quad 0 \quad \cdots \quad 0]$. This formulation works well for even time-varying reference signals unlike most other formulations, which assume a steady state fixed set point as the reference signal. However, the problem with this formulation and MPC in general, is that one requires knowledge of the future "N" values of the disturbances $\underset{\rightarrow}{d}$ in order to use the MPC method exactly. In many cases, when a linear model for the disturbance exists, one may implement it in the formulation and use it to derive the MPC controller as shown above. But in cases, when a disturbance model is not available, one may have to use some kind of disturbance estimation in Equation (21). In the case of the dual-stage actuation system in this document, the procedure required to arrive at a valid linear model for nonlinearities like hysteresis is nontrivial. Hence, it is not possible to estimate the exact disturbance in the system accurately. A simple way of overcoming this difficulty is to obtain the previous value of the disturbance and assume that the disturbance remains the same for the next N steps, which is basically an extended zero-order estimate. In this case, we replace $\underset{\rightarrow}{d}$ by its estimate

$$\underset{\rightarrow}{d}_{est} = \begin{pmatrix} d_{k-1} & d_{k-1} & \cdots & d_{k-1} \end{pmatrix}^T \quad (23)$$

where $d_{k-1} = X_k - AX_{k-1} - BU_{k-1}$. The actual error between the actual and estimated values of d_{k+N-1} is given by

$$d_{k+N-1} - d_{k-1} = (d_{k+N-1} - d_{k+N-2}) + (d_{k+N-2} - d_{k+N-3}) + \cdots (d_k - d_{k-1}). \quad (24)$$

Considering one general term on the right hand side of the above equation, we have

$$d_{k+j} - d_{k+j-1} = \int_0^T e^{A_c\tau}(d((k+j)T + T - \tau) - d((k+j)T - \tau))\mathrm{d}\tau \quad (25)$$

Now since

$$(f(b) - f(a)) = \int_a^b \mathrm{d}f = \int_a^b \frac{\mathrm{d}f}{\mathrm{d}\theta}\mathrm{d}\theta,$$

we write

$$d_{k+j} - d_{k+j-1} = \int_0^T e^{A_c\tau}\left(\int_{(k+j)T-\tau}^{(k+j)T+T-\tau} \dot{\mathrm{d}}(\theta)d\theta\right)\mathrm{d}\tau \quad (26)$$

where "θ" is a dummy variable. This error is $O(T^2)$ if $d(t)$ and its time-derivative $\dot{d}(t)$ are bounded. The total error in the estimation of the disturbance can be given by

$$d_{k+N-1} - d_{k-1} = N \cdot O(T^2) = O(T^2) \quad (27)$$

if $N \ll 1/T$, which is assumed and is valid for this system, $N = 10$, $T \leq 1$ms is used.

The MPC controller is then designed as discussed in detail above. The final control law is exactly given by Equations (21) and (22). We also saturate the control effort by limiting the controller values to lie between its maximum and minimum values (Table 1).

EXPERIMENTAL TESTING AND RESULTS

A simple experimental setup is built following the design principles discussed in detail in this section.

Table 1. Values for appropriate constants for the system.

Symbol	Description	Value
ω_n	Natural frequency	1.113e3 rad/s
ξ	Damping ratio	0.75
β	Constant in B matrix	0.132 m/V.s^2
λ	Control weight in MPC	Varied as shown in results
U_{max}	Maximum control voltage	1000 V
U_{min}	Minimum control voltage	−200 V

Figure 7 shows the simple setup. The setup consists of a controllable DC motor capable of producing large enough torque to provide the necessary preload driving a lead screw-nut assembly via a coupling and an axial bearing. The nut is connected to a piezoelectric actuator capable of producing 12.5 kN of blocked force and 105 μm free displacement for a voltage range of −200 to 1000 V. The free end of the piezoelectric actuator is connected to a brake pad, which pushes against a load cell at the end of the setup. A high voltage power supply is used to power the Piezoelectric actuator. A DSpace controller board is used to interface Simulink to the system during the real-time control.

Model Verification

A simple step response test is performed to check if the system response follows that of a second order system. Once a preload of around 1.25 kN is achieved using the DC motor, the piezoactuator is given a step voltage input. This is repeated using a slow square wave voltage input and the resulting generated force response is recorded. The voltage input is stepped up and down repeatedly between −200 and 900 V. Figure 8 shows the generated force response.

It is evident that the system follows the response of a second order system with both overshoot and oscillations. Based on this data, the values of the natural frequency and damping ratio of the second order system are assigned Table 1. Also the open-loop hysteresis curves for the system are determined by applying a sinusoidal voltage to track a sinusoidal reference force value. Figure 9 shows the resulting open-loop hysteresis curves. It is seen that the large hysteresis effects give rise to poor tracking performance. The major goal of the controller design is to minimize the hysteresis effects

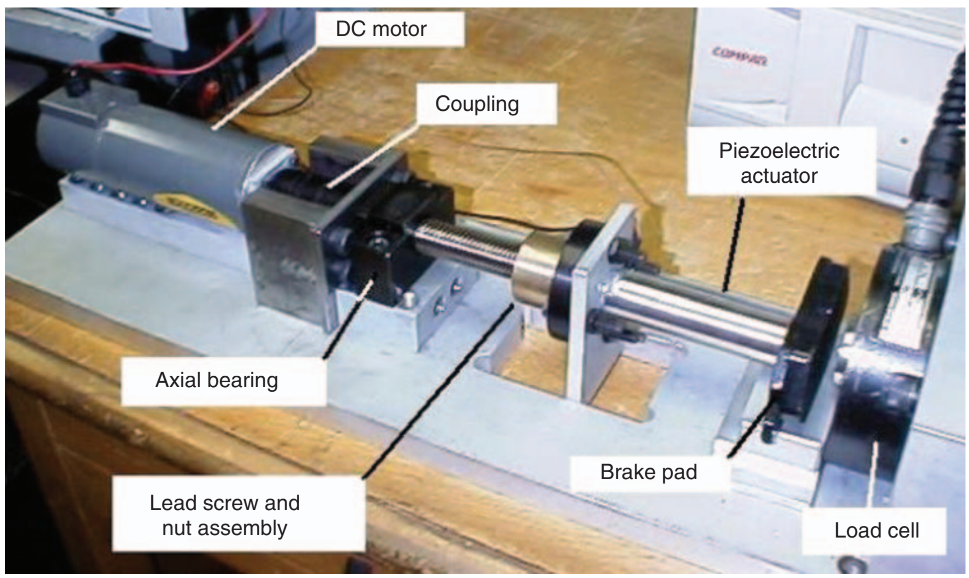

Figure 7. *Experimental setup.*

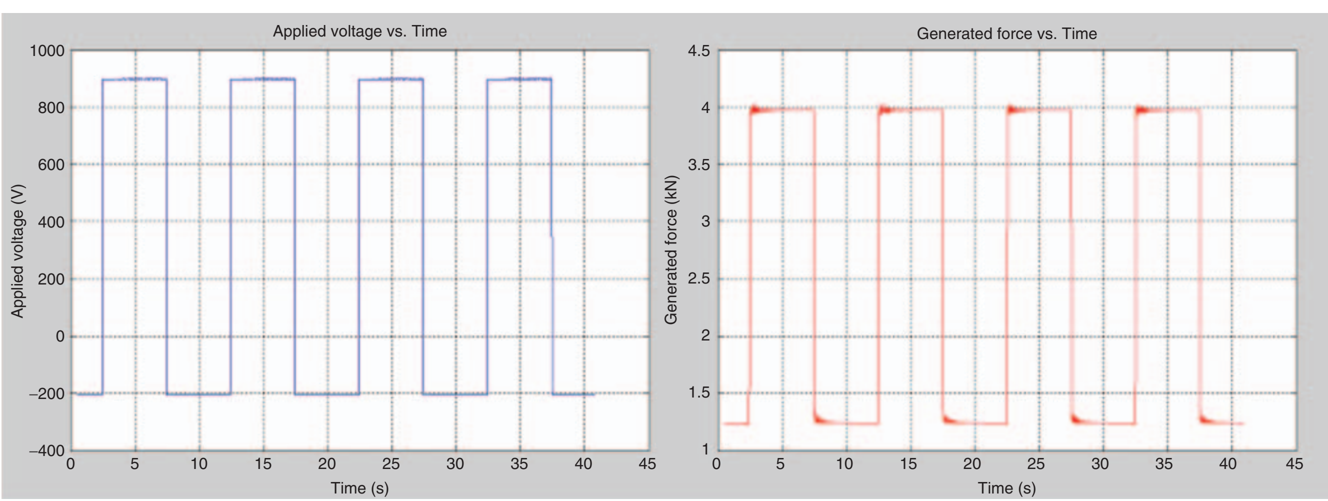

Figure 8. *Step response of system in force phase using piezoelectric actuator.*

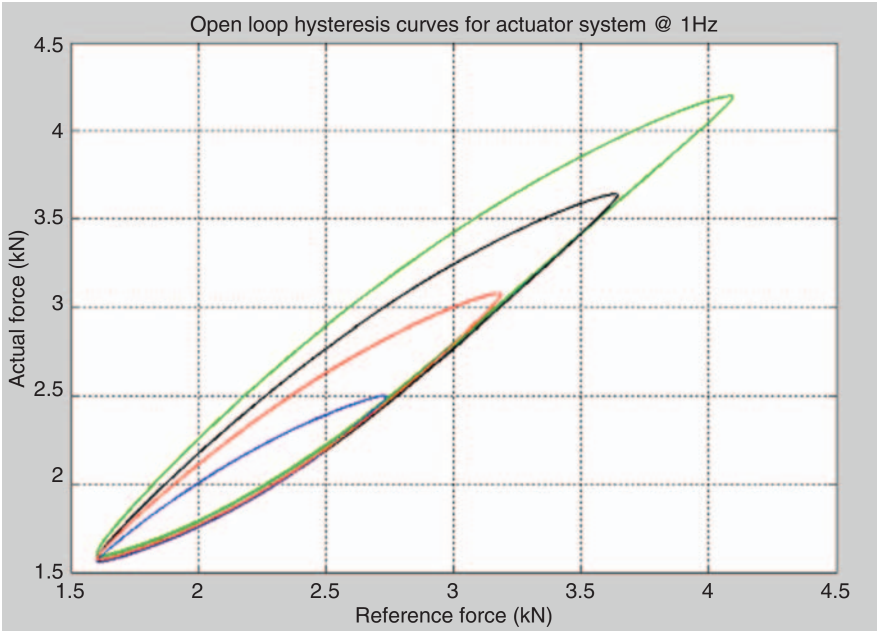

Figure 9. *Open loop hysteresis curves in force phase.*

thereby linearizing the closed loop performance without compromising the stability requirement. Another important point to be noted is that the preload from the first phase of the process using the DC motor must be high enough to make the mechanism meet the force requirement. This can be avoided using a longer piezoactuator with extended stroke capabilities.

Control Law Implementation and Testing

The model predictive controller designed in the previous section is applied to the force phase of the actuation process of the system. The controller is designed to force the system to track different reference signals like step and sinusoidal signals at different frequencies. Figures 10–16 show the tracking results for different reference inputs to the system. It is noted that the figures show the results of the controller after the first phase of the actuation system is completed. From the figures, it is observed that the model predictive controller provides excellent tracking performance. Different values for the parameter "λ" in the model predictive controller, which is the weight on the control changes in the cost function J, are chosen to illustrate the effect of choosing different weighting on control effort changes. For step reference tracking shown in Figure 10 and 11, the response time decreases as the value of "λ" decreases and though the lower value of "λ" means faster response and settling times, it is observed that that for "λ" $= 0.00025$, overshoot is present in the system response. This is because the error in the disturbance estimate is high when very fast response times are desired. However, a response time of less than 0.1 s is very desirable and hence the system response for $\lambda = 0.0005$ is excellent. Figures 12–15 show the tracking response for sinusoidal reference inputs at two different frequencies, namely 1 Hz and 10 Hz. Again, it is noted that the steady state tracking performance improves as the "λ" is decreased. However, the transient performance deteriorates as "λ" is decreased, because at lower "λ" the weight on the control effort change is low, which allows faster and higher changes in the control input. This leads to undesirable transient performance since the error in the disturbance estimates becomes higher due to higher and quicker control changes. This also highlights the classical trade-off in control system design between transient and steady state performances. However, MPC is preferred to a simple PID controller since the PID controller does not account for disturbances explicitly in its structure. Moreover, a PID controller will need to be

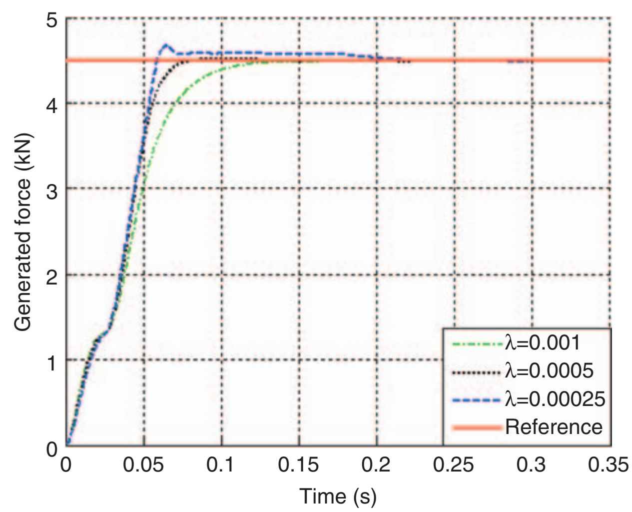

Figure 10. *Step reference tracking – control force vs. time for various values of "λ".*

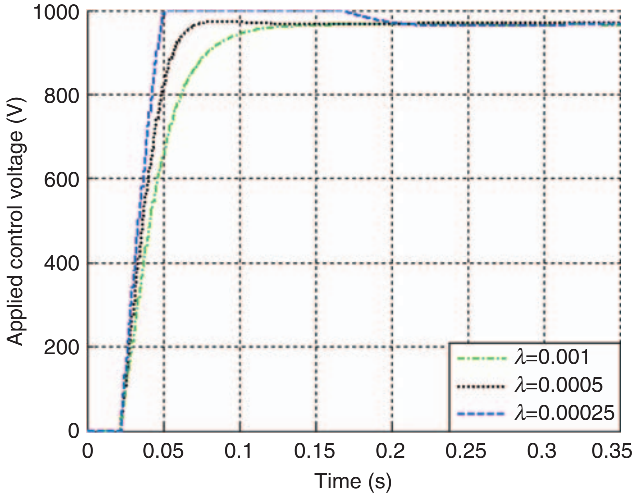

Figure 11. *Step reference tracking – control voltage vs. Time for various values of "λ".*

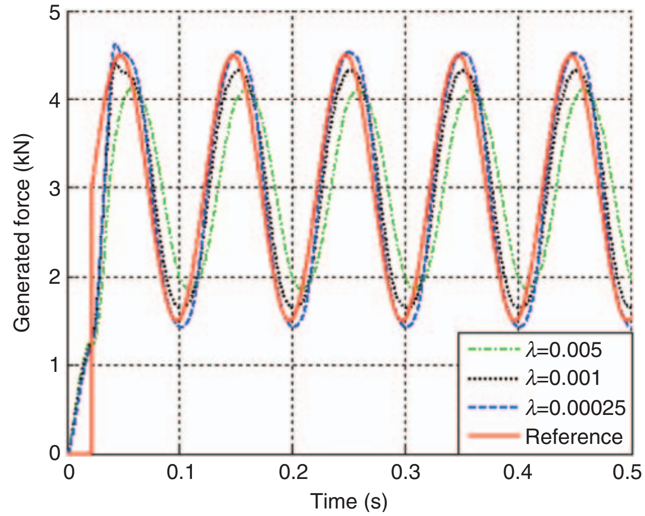

Figure 14. *Sinusoidal Reference (10 Hz) tracking – generated force vs. time for various values of "λ".*

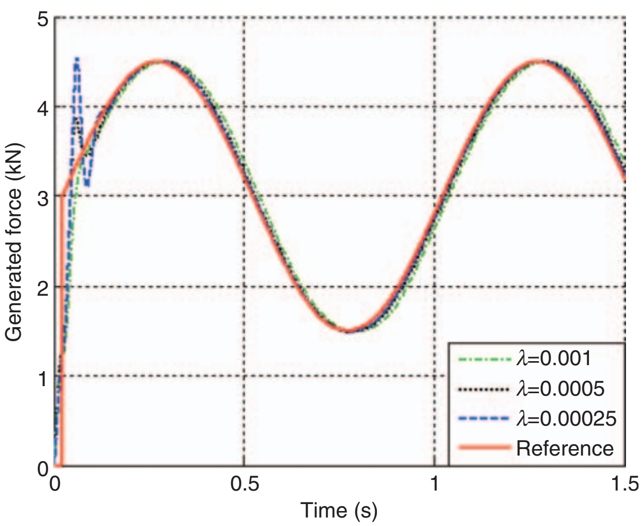

Figure 12. *Sinusoidal reference (1 Hz) tracking – generated force vs. time for various values of "λ".*

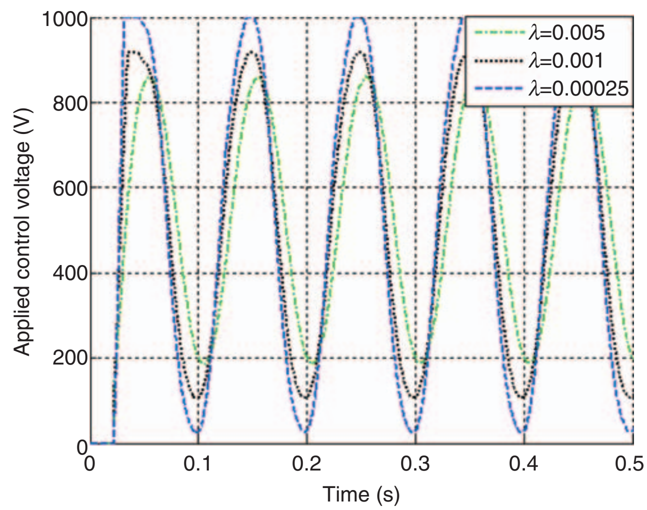

Figure 15. *Sinusoidal reference (10 Hz) tracking – control voltage vs. time for various values of "λ".*

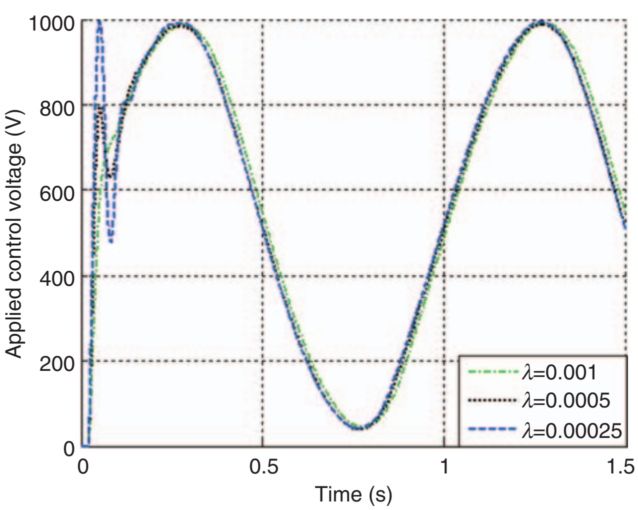

Figure 13. *Sinusoidal reference (1 Hz) tracking – control voltage vs. time for various values of "λ".*

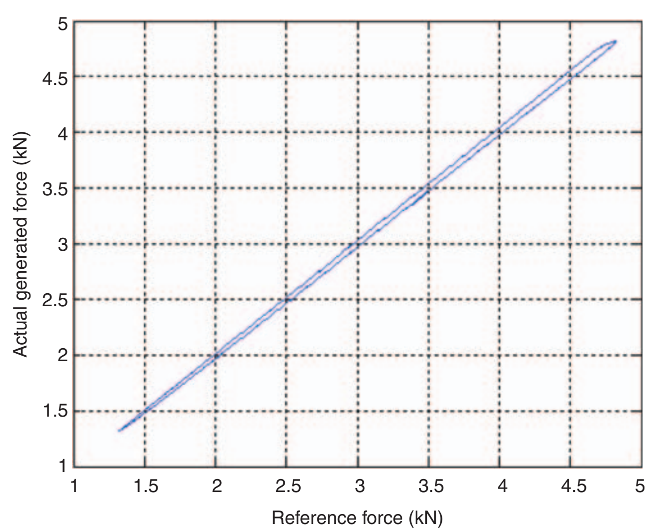

Figure 16. *Closed loop relation between reference and actual force.*

retuned frequently at different operating conditions to provide a consistent performance, while no frequent tuning is required for MPC. For sinusoidal reference tracking inputs, the controller performance deteriorates as the frequency increases. This again may be attributed to the accuracy of the disturbance estimates at high frequencies. However, based on the overall performance "λ" = 0.0005 may be chosen as the best value for the model predictive controller among the various values shown. Figure 16 shows the steady state closed loop relation between the reference and actual force for "λ" = 0.0005 for a simple sinusoidal voltage at 10 Hz.

CONCLUSIONS

A new two-stage hybrid actuation mechanism is presented. The operating principle basically involves the separating the actuation process into two stages with contrasting force and stroke requirements. The paper discusses the possible designs using this concept. A design methodology along with the operating strategy of the two-stage mechanism is presented. A robust controller is designed based on the MPC concept. An extended one-step delayed disturbance estimate is used to account for nonlinearities in the system like the hysteresis and other internal and external disturbances. The results show that this MPC controller is an excellent candidate for controlling piezoelectric actuator systems and may be extended to control other smart material systems like magnetostrictive material devices and MR fluid devices. Certain drawbacks of MPC include (i) the requirement of the knowledge of future reference inputs that the systems is required to track and (ii) the inability to directly relate the system performance like settling time and overshoot with respect to the controller parameter "λ". While the knowledge of future reference inputs are known in many applications, it is not readily available in some real applications, where the reference inputs are stochastic and time-varying. Hence, there is a need for better controllers that can eliminate these drawbacks. However, it is still very desirable to utilize the advantages of the concept of MPC in developing new control strategies while trying to eliminate its drawbacks. Future work will be centered on Model Predictive Sliding Mode control (MPSMC). This idea is focused on merging the popular concepts of MPC and SMC (Sliding Mode Control) in developing a new concept that essentially eliminates the drawbacks of both MPC and SMC individually while simultaneously taking advantage of their benefits (Young et al., 1999 and Utkin et al., 1999). This controller, fundamentally a sliding mode controller, is designed to enforce the sliding mode in a smooth optimal manner, wherein the optimization is carried out using the MPC concept. This will provide a means of choosing the system eigenvalues that define the system performance like settling time while eliminating the well-known phenomenon of chattering and achieving system stability using the MPC concept.

ACKNOWLEDGMENTS

The authors wish to thank the Powertrain Systems Research lab at GM R&D, Warren, Michigan for providing the funding for this research.

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