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Physics of Currents and Potentials II. Classical Singlet-Triplet Electroweak Theory with Non-point Particles

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Abstract: The formulation of classical singlet-triplet (electroweak) theory, in which point particles are replaced by continual current fields, has been presented. The expression for the Lagrangian of the theory and complementary algebraic and differential constraints, imposed on the currents of the theory, have been suggested. Classification of stationary and wave states for the singlet-triplet theory has been given. Stationary states correspond to massive particles with the current zone of a finite volume. The theory contains a number of wave states, both one-sector (singlet or triplet waves) and compound two-sector ones (singlet-triplet waves). Wave states differ in number of currents: zero-current waves (free singlet or free triplet waves), one-current, two-current, three-current and four-current ones. Wave states also differ in character of four-dimensional wave vector (the waves with time-like and space-like wave vector). Some forms of waves may have negative density of energy. Some wave states can be treated as classical models of a neutrino. Neutrino states are classified in accordance with the character of the current which forms the state: singlet (maxwellian) neutrino, Yang-Mills triplet neutrino, Maxwell-Yang-Mills singlet-triplet neutrino.

Keywords: Lagrangian; Singlet-Triplet Theory; Classification of the States in the Singlet-Triplet Theory; Yang-Mills Triplet Waves; Classical Models of Neutrino
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1. Introduction

Let us extend the approach stated in the article [1] over the unified theory of electroweak interaction. Within the framework of this extension we should postulate the existence of two sectors of physics of currents and potentials: singlet (or maxwellian) sector and triplet (or Yang-Mills) sector. Each sector contains a dyad "current/potential". Singlet

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sector contains singlet (maxwellian) current J^{ν} and singlet (maxwellian) potential W^{ν} which is dynamically conjugated with it. Triplet sector contains Yang-Mills triplet of currents $\mathbf{J}^{\nu} = \left\{ \overset{a}{\mathrm{J}}{}^{\nu} \right\}$ and dynamically conjugated with it Yang-Mills triplet of potentials $\mathbf{W}^{\nu} = \left\{ \overset{a}{\mathrm{W}}{}^{\nu} \right\}$. Over-letter indices a, b, c number the components of Yang-Mills triplets: $a = 1, 2, 3$. Greek indices $\mu, \nu, \ldots$ number Lorentz components of 4-vectors. They run over values 0, 1, 2, 3.

The adjectives "singlet" and "maxwellian", which are synonyms and often appear in the text, will be sometimes expressed by symbols S- or M-, for example, "S-current" or "M-current" instead of the term "singlet current". Generally speaking, the terms "triplet" and "Yang-Mills" are not synonyms: octuplet (chromodynamical) sector of physics deals with Yang-Mills octuplet of currents and dynamically conjugated with it Yang-Mills octuplet of potentials. However, within the framework of the present article, which is devoted to classical electroweak theory, we shall treat the terms "triplet" and "Yang-Mills" as synonyms, and if necessary, we shall denote them with symbols T- or YM-, for example, "T-current" or "YM-current" instead of the term "triplet current".

YM-geometry is Euclidian: there is no necessity to introduce contravariant and covariant indices. YM-vectors are denoted with bold type in indexless notation. In some cases we have to use bold type for indexless notation of the three space components of Lorentz 4-vectors. We hope that the reader will be able to tell these objects from YM-vectors.

Twice repeating YM-indices imply summation, such as, for example, scalar YM-product of YM-vectors $\mathbf{J}^{\nu}$ and $\mathbf{W}^{\nu}$:

$$\mathbf{J}^{\mu} \cdot \mathbf{W}^{\nu} \equiv \overset{a}{\mathrm{J}}{}^{\mu} \overset{a}{\mathrm{W}}{}^{\nu} \equiv \sum_{a=1}^{3} \overset{a}{\mathrm{J}}{}^{\mu} \overset{a}{\mathrm{W}}{}^{\nu}.$$

In the three-dimensional space of YM-vectors it is possible to enter a vector product through Levi-Civita 3-symbol $\overset{abc}{\varepsilon}$ in a conventional way: if $\mathbf{C}^{\mu\nu} = \mathbf{A}^{\mu} \times \mathbf{B}^{\nu}$,

$$\overset{a}{\mathrm{C}}{}^{\mu\nu} = \overset{abc}{\varepsilon} \overset{b}{\mathrm{A}}{}^{\mu} \overset{c}{\mathrm{B}}{}^{\nu},$$

while object $\overset{a}{\mathrm{C}}{}^{\mu\nu}$ is Yang-Mills 3-vector and Lorentz tensor of the second rank in Minkovski four-dimensional space-time.

Singlet M-potential W^{ν} does not coincide with the electromagnetic potential A^{ν} considered in the article [1]. Singlet M-current J^{ν} also does not coincide with the electromagnetic current which we will denote as $\underset{em}{\mathrm{J}}{}^{\nu}$. However, as we will show later, in the "mixed" Maxwell-Yang-Mills' singlet-triplet world there are "pure" maxwellian (singlet) states in which electromagnetic potential A^{ν}, proportional to singlet potential W^{ν}, and electromagnetic current, proportional to singlet current J^{ν}, can be introduced to describe a state, thereby we will revert to the classical electromagnetic theory of non-point particles, formulated in the article [1]. With description of an arbitrary mixed singlet-triplet state, such extraction of electromagnetic potential in the form of some linear combination of singlet and triplet potential is also possible, but it is irrational: it overloads simple and

natural algebra of YM-vectors.
Dynamics of the two field dyads "currents/potentials" of the singlet-triplet world is derived from the principle of the least action at minimization of action functional S:

$$S = \int L' \mathrm{d}\Omega. \tag{1}$$

The total space-time integration is implied in this expression; L' is the effective Lagrangian consisting of the sum of the base Lagrangian L and the addition to the Lagrangian L_{ad}, which is caused by the necessity of accounting differential and algebraic constraints, which are a priori imposed on the four currents $(\mathrm{J}^\nu, \mathbf{J}^\nu)$.
The base Lagrangian$L$ consists of the sum of two sector Lagrangians - the singlet Lagrangian L_S and the triplet Lagrangian L_T:

$$L = L_S + L_T. \tag{2}$$

The relation (2) means that singlet and triplet sectors of physics do not intermingle with each other on the level of the base Lagrangian.
Each of the two sector terms of the base Lagrangian (2), in its turn, consists of three terms, which correspondently describe the current part of the Lagrangian L_{cur}, the Lagrangian of current and potential L_{int} interaction, and the free field Lagrangian L_f:

$$\begin{aligned} L_S &= L_{S,cur} + L_{S,int} + L_{S,f}, \\ L_T &= L_{T,cur} + L_{T,int} + L_{T,f}. \end{aligned} \tag{3}$$

Integrals in (1), containing the Lagrangians of free fields, are taken from all over the 4-space. Integrals in (1), containing the current Lagrangian L_{cur} and the interaction Lagrangian L_{int} are taken over current 4-zones Ω_J, in which at least one of the four currents is nonzero. It is supposed that in the current zones each of the four currents is space-like:

$$\mathrm{J}^\nu \mathrm{J}_\nu \le 0, \tag{4}$$

$$\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}{}_\nu \le 0; \overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}{}_\nu \le 0; \overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}{}_\nu \le 0. \tag{5}$$

The condition of form (4) was substantiated for electromagnetic current in the previous article [1]. Current inequalities (4) and (5) are taken here by analogy with the inequality for $\underset{em}{\mathrm{J}}{}^\nu$ as the postulates of physics without which the construction of the classical theory of non-point particles is impossible.
At the three-dimensional boundaries σ_J of the four-dimensional current zones Ω_J at least one of the currents becomes isotropic, and, correspondingly, the sign of equality is reached at least in one of the relations (4). Such boundary, termed in the article [1] a "pomerium", can be common for some currents or can disintegrate into different boundaries for different currents. Beyond such boundary, on which isotropization of one of the four currents took place, this current is identically zero. At the boundary itself, this isotropic current is orthogonal to 4-vector of normal to the boundary n^μ.
According to the reasons given in the article [1], we have to accept that pseudo-Euclidian

modules of space-like currents are limited by absolute value. Thereby, we must postulate the existence of such fundamental constant j_S, that

$$-\mathrm{J}^{\nu}\mathrm{J}_{\nu} \leq \mathrm{j}_S^2 \tag{6}$$

and of such fundamental constant j_T, that

$$\begin{aligned} &-\overset{1}{\mathrm{J}}{}^{\nu}\,\overset{1}{\mathrm{J}}{}_{\nu} \leq \mathrm{j}_T^2;\\ &-\overset{2}{\mathrm{J}}{}^{\nu}\,\overset{2}{\mathrm{J}}{}_{\nu} \leq \mathrm{j}_T^2;\\ &-\overset{3}{\mathrm{J}}{}^{\nu}\,\overset{3}{\mathrm{J}}{}_{\nu} \leq \mathrm{j}_T^2. \end{aligned} \tag{7}$$

It is unlikely that God would have been so wasteful to have created two different fundamental constants of the same dimension, and the equality $\mathrm{j}_S = \mathrm{j}_T$ does not seem impossible.
The reasons, given below in p.3.4, make it possible to come to conclusion that $\mathrm{j}_T \leq \mathrm{j}_S{}^2$.
Adoption of inequalities (6) and (7) is necessary for obtaining finite mass of particles within the framework of the classical theory under consideration.
The equality sign in relations (6) and (7) is reached at some three-dimensional boundaries of the four-dimensional current zones. Some cavitated no-current tubes are limited by these boundaries inside the current zone. Let us call such cavitated no-current tube with a term "***latebra***". This word was translated from the language of the ancient Romans as a "sheltered place", "hiding place" and it also had the meaning "inner cavity". The boundary of a cavitated tube will be termed "***latens***" (hidden, invisible).
According to the reasons provided in the article [1], it can be expected that diametrical dimensions of ***latebra*** are of the order of Planckian length r_p. The longitudinal dimension of ***latebra*** is determined by the appearing in the theory unknown fundamental constant of length dimension r_0: $r_0 > r_\mathrm{p}$.
Latebra and ***latens*** are inaccessible in any laboratory experiment – as one of the few survived Heraclitus' fragments reads, "nature likes hiding". Late Roman commentator Macrobius, while interpreting this idea one thousand years after Heraclitus, said that "nature feels disgusted to be exposed open and naked". However, to give sense to the classical theory of currents and potentials discussed here, we must consider currents and potentials as measurable physical quantities even in case if the only device, which had already been known to Isaac Newton, Sensorium Dei, is applicable for their measuring.
According to the reasons given in the article [1], when constructing the theory of non-point particles, we should take into account Riemannian space-time curvature, generated by large quantities of energy-momentum tensor components in the region occupied by currents and inside the cavitated no-current ***latebrae***.
As in the article [1], we will suppose that this account of curvature can be provided according to Einsteinian recipe:

- obtain the field theory equations in Minkowski coordinates, ignoring the space-time curvature;

[2] See the form below (292), which determines the relation between the constants j_S and j_T.

- rewrite the obtained equations in arbitrary curvilinear coordinates with arbitrary metric tensor which possesses Minkowski signature $(+- - -)$ at each point of the four-dimensional space-time continuum;
- subordinate the metric tensor to Einstein equations, which read that the Ricci tensor is proportional to energy-momentum tensor; the proportionality coefficient is determined by gravitation constant.

When solving the wave states problems, where the boundaries of pomerium and latens are missing, the space-time curvature can be ignored, supposing that energy density in the wave is insufficient for forming substantial deviations from Minkowski plane geometry.

The singlet-triplet theory, constructed here, assumes the existence of the states where the conditions of isotropy of currents (4) or (5) for one or a few currents are satisfied in some four-dimensional zone Ω_N rather than on the three-dimensional hyper-surface of pomerium σ_{J}. We shall name these states "neutrino states"; the corresponding isotropic currents will be named "neutrino currents". Neutrino states should be treated as non-stationary, transitional states generated by some instability of "neutrinoless" states. However, the theory also admits the existence of the model wave neutrino states, in which neutrino zone Ω_N coincides with the whole four-dimensional space. While considering such states, we have to sacrifice the condition of currents and fields disappearance "at infinity". Generally speaking, these conditions must be always supposed to be satisfied at minimization of the action functional (1).

Summing up the "Introduction", we would like to remind the reader that the constructed theory, as in the article [1], includes the unknown fundamental dimension constant r_0. This constant is further assumed as a length unit. Velocity of light c is also assumed as a unit. Constants r_0 and c do not explicitly appear in the theory formulas.

2. The Base Lagrangian of the Singlet-triplet Theory

2.1 The Interaction Lagrangian. The Weinberg Angle

Let us suppose, as in classical electrodynamics, that the singlet interaction Lagrangian $L_{S,int}$ is proportional to scalar product of singlet current J^ν and singlet potential W^ν:

$$L_{S,int} \sim \mathrm{J}^\nu \mathrm{W}_\nu, \tag{8}$$

and, similarly, the triplet interaction Lagrangian $L_{T,int}$ is proportional to "twice scalar" product of the triplet current $\mathbf{J}^\nu$ and the triplet potential $\mathbf{W}^\nu$:

$$L_{T,int} \sim \mathbf{J}^\nu \cdot \mathbf{W}_\nu. \tag{9}$$

The expression (9) implies summation by unwritten explicitly YM-indices and summation by Lorentz indices (Euclidean scalar product in three-dimensional Euclidean space of YM-vectors and pseudo-Euclidean scalar product of 4-vectors in four-dimensional pseudo-Euclidean Minkowski space-time).

Proportionality coefficients in formulas (8) and (9) are negative [2]. We do not have any a priori grounds to suggest that the expressions (8) and (9) must enter into the general interaction Lagrangian with identical weight. We shall assume that singlet and triplet contributions to L_{int} have different weight factors p_S and p_T:

$$L_{int} = -\frac{1}{2p_S}\mathrm{J}^{\nu}\mathrm{W}_{\nu} - \frac{1}{2p_T}\mathbf{J}^{\nu}\cdot\mathbf{W}_{\nu}. \quad (10)$$

$(p_S > 0; p_T > 0)$.
In the units of measurement used here, currents and potentials have an electric charge dimension. The Lagrangian and the action functional have a dimension of an electric charge square. The singlet interaction constant of p_S and triplet interaction constant p_T are dimensionless, although their numerical values are determined by selecting an electric charge unit of measurement (as it is possible to introduce an arbitrary coefficient of proportionality between the left and the right parts of the relation (1), which determine the action functional). It is convenient to normalize the sum of squares of constants p_S andp_T per unit:

$$p_S^2 + p_T^2 = 1. \quad (11)$$

The relation (11), implying a definite selection of an electric charge unit of measurement, by normalizing the general contribution of the interaction Lagrangian (10) to the total Lagrangian, keeps us from a temptation to consider the model "one-sector" worlds: the singlet world with the triplet sector ($p_T \to \infty$) "switched off" or the triplet world with the singlet sector ($p_S \to \infty$) "switched off". The relation (11) allows us to use one constant instead of two fundamental constants p_S and p_T. If we assume that $p_T = \sin\theta_w$ $(0 < \theta_w < \frac{\pi}{2})$, according to (11) we should assume that $p_S = \cos\theta_w$.
The fundamental constant θ_w is usually called Weinberg angle. It would be more correct (but more ponderous) to call this constant "the angle of the singlet-triplet mixing" and, correspondingly, to denote it with θ_{ST}.
The same electric charge unit that was used in (11) for Weinberg parameters normalizing, is used to measure currents and potentials. Accordingly, further the currents and potentials are considered dimensionless.

2.2 The Current Lagrangian

Based on the results of the preceding article [1], we shall assume that the contribution of each current to the base Lagrangian in neutrinoless state is proportional to pseudo-Euclidean square of current. "Assigning" corresponding weight factors, we shall write the current part of the Lagrangian as follows:

$$L_{cur} = -\frac{1}{8p_S^2}\mathrm{J}^{\nu}\mathrm{J}_{\nu} - \frac{1}{8p_T^2}\mathbf{J}^{\nu}\cdot\mathbf{J}_{\nu}. \quad (12)$$

The weight factor in the singlet part of the current Lagrangian (12) is set on the basis of compliance with the electromagnetic theory of non-point particles, constructed in the preceding article [1]. Weight factor in the triplet part of the current Lagrangian (12) is

set on the basis of current "singlet-triplet equality", by means of substitution of $p_S \to p_T$, $\mathrm{J}^\nu \to \mathbf{J}^\nu$ in the singlet part of the current Lagrangian.
In the state which is neutrino by some current, the pseudo-Euclidean square of this current in the neutrino 4-zone $\mathbf{\Omega}_N$ is equal to zero, and the contribution of this current to the current Lagrangian is missing (12). However, the pseudo-Euclidean square of a neutrino current appears in the additional Lagrangian L_{ad} with an arbitrary Lagrange's multiplier.

2.3 The Field Lagrangian

The field Lagrangian in (2) and (3) is constructed in the form of the expression which is quadratic by tensors of the singlet and triplet field $\mathrm{W}_{\mu\nu}$ and $\mathbf{W}_{\mu\nu}$ with identical weight factors:

$$L_f = -\frac{1}{16\pi}\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} - \frac{1}{16\pi}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}. \tag{13}$$

The singlet field tensor $\mathrm{W}_{\mu\nu}$ is constructed in the same way as it is constructed in classical electrodynamics [2]

$$\mathrm{W}_{\mu\nu} = \partial_\mu \mathrm{W}_\nu - \partial_\nu \mathrm{W}_\mu. \tag{14}$$

Yang-Mills triplet field tensor was constructed in the classical article of these two authors [3][3]. In the symbols we have accepted, it looks as follows:

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + p_T \mathbf{W}_\mu \times \mathbf{W}_\nu, \tag{15}$$

or, with explicitly specified YM-indices:

$$\overset{a}{\mathrm{W}}_{\mu\nu}= \partial_\mu \overset{a}{\mathrm{W}}_\nu - \partial_\nu \overset{a}{\mathrm{W}}_\mu + p_T \overset{abc}{\varepsilon} \overset{b}{\mathrm{W}}_\mu \overset{c}{\mathrm{W}}_\nu . \tag{16}$$

The choice of expressions (15), (16) for YM-field tensor was in due time motivated by considerations of SU(2) – gauge invariance (in the era of Yang-Mills they spoke of an "isotopic gauge invariance" [3]). However, the explicit construction of YM-potential and YM-field tensor , in this way, requires appealing to the notion "wave function phase" of point particles and it is inadmissible in the classical theory of non-point charges, which is developed here. Within the framework of this classical theory it is reasonable to treat the potential and current as primary, irredundant and non-interpreted concepts, and to treat YM-field tensor (15) (in the spirit of "pedagogical" motivations typical for the course of Landau and Lifshitz [2]), as the simplest antisymmetric Lorentz second-rank tensor (and, at the same time, YM-vector), which can be constructed from YM-vector.
Fixation of the multiplicative constant in (15) before the nonlinear term suggests a certain choice of measurement unit for the electric charge. Change of the unit charge in (15) requires re-scaling of this multiplier and violates unit coefficient normalization (11). The

[3] Wolfgang Pauli, as pointed by N. Straumann [4], developed the main aspects and formulas known now as "Yang-Mills theory" already in summer 1953, but he didn't publish his results.

fact that this unit of charge measurement, connecting (11) and (15), coincides with the charge value of the electron, can be treated as a casual gift[4].

2.4 The Total Base Lagrangian of the Singlet-triplet Theory

By substituting the written out expressions for separate terms of the base Lagrangian (10), (12), (13) into (2) and (3), we obtain the total base Lagrangian of a neutrinoless state:

$$L = -\frac{1}{8p_S^2}\mathrm{J}^\nu \mathrm{J}_\nu - \frac{1}{8p_T^2}\mathbf{J}^\nu \mathbf{J}_\nu - \frac{1}{2p_S}\mathrm{J}^\nu \mathrm{W}_\nu - \frac{1}{2p_T}\mathbf{J}^\nu \cdot \mathbf{W}_\nu - \frac{1}{16\pi}\left(\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} + \mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}\right). \quad (17)$$

In this Lagrangian the singlet field tensor $\mathrm{W}_{\mu\nu}$ is calculated by the formula (14) through the components of the singlet potential W_μ. The triplet field tensor $\mathbf{W}_{\mu\nu}$ is calculated through the components of the triplet potential $\mathbf{W}_\mu$ by formula (15). Weight factors p_S and p_T ("Weinberg parameters") are expressed by the formulas mentioned in p. 2.1 through the fundamental constant θ_w which is exterior for this theory. Currents J^ν and $\mathbf{J}_\nu$, potentials W^ν and $\mathbf{W}^\nu$ are primary, non-interpreted variables of the theory, which are inexpressible by other more elementary physical concepts. If, in accordance with the specific initial conditions, the formation of the neutrino 4-zone $\mathbf{\Omega}_N$, occupied by isotropic current, is possible in the singlet-triplet problem, within this zone the contribution of a corresponding neutrino (isotropic) current falls out of the two first terms in the base Lagrangian (17), but this contribution is restored (with an arbitrary Lagrange's multiplier) in the additional Lagrangian L_{ad}, since the condition of current isotropy must be considered as holonomic constraint imposed on the solution of the singlet-triplet problem.

3. The Algebraic and Differential Constraints for Currents. The Additional Lagrangian

3.1 Differential Condition for Singlet Current Conservation

As in the classical electrodynamics [1], the condition for S-current conservation is adopted in the singlet sector of the theory:

$$\partial_\mu \mathrm{J}^\mu = 0. \quad (18)$$

The condition (18) requires appearing of the term in the form of $\frac{1}{2p_S^2}\chi\partial_\nu \mathrm{J}^\nu$ in the additional Lagrangian L_{ad}, where χ is the arbitrary Lagrange's multiplier depending on the space-time coordinates x_μ, and the multiplier $\frac{1}{2p_S^2}$ is introduced for the convenience of the additional and base Lagrangian combination. By extracting from this term the total

[4] This "gift" faintly signifies that the singlet-triplet theory is possibly not a final and unimprovable version of physics of electroweak interactions.

4-divergence $\frac{1}{2p_S^2}\partial_\nu(\chi \mathrm{J}^\nu)$, which disappears after the Lagrangian integration over the entire 4-space, we can denote the corresponding contribution to the additional Lagrangian as follows

$$L_{ad(\chi)} = -\frac{1}{2p_S^2}\mathrm{J}^\nu \partial_\nu \chi.$$

As it is known, the differential constraint (18) is ***natural*** for the Lagrangian (17). This constraint is the result of Maxwell field equations, appearing with minimization of the action functional with the Lagrangian (17) under any Lagrange's multiplier χ. So, it would be reasonable to assume from the outset that $\chi = 0$ and not to include the term $L_{ad(\chi)}$ into the additional Lagrangian.

3.2 Differential Condition for the Triplet Current

Generally speaking, the triplet YM-current, by contrast to the singlet M-current, is not conserved. Differential constraint, imposed on YM-current [3], can be described in the form of the relation which we are free to adopt as a postulate:

$$\partial_\mu \mathbf{J}^\mu + p_T \mathbf{W}_\mu \times \mathbf{J}^\mu = 0. \tag{19}$$

Multiplying the equation (19) by the arbitrary Lagrange's multiplier $\boldsymbol{\chi}$ (Lorentz scalar and YM-vector), after the extraction of total 4-divergence, we obtain a corresponding contribution to the additional Lagrangian coupled with the Lagrange's multiplier $\boldsymbol{\chi}$:

$$L_{ad}(\chi) = \frac{1}{2p_T^2}\left(-\mathbf{J}^\mu \partial^\mu \boldsymbol{\chi} + p_T \boldsymbol{\chi} \cdot (\mathbf{W}_\mu \times \mathbf{J}^\mu)\right).$$

In this expression the multiplicative constant $\frac{1}{2p_T^2}$ is introduced for the convenience of combination of the base Lagrangian and the Lagrangian $L_{ad}(\chi)$.
By varying the total Lagrangian L with addition of $L_{ad}(\chi)$ it is not difficult to establish that the differential constraint (19) for YM-current is the result of Yang-Mills field equations with any option of the Lagrange's multiplier $\boldsymbol{\chi}$ which satisfies the condition

$$\partial_\mu \boldsymbol{\chi} + p_T \mathbf{W}_\mu \times \boldsymbol{\chi} = 0.$$

Supposing that $\boldsymbol{\chi} = 0$, we are satisfying this condition. Such option of $\boldsymbol{\chi}$ may be not convenient enough to solve some problems, but its use is acceptable in any problem. Consequently, we may avoid including term $L_{ad}(\chi)$ into the additional Lagrangian.
Let us call the choice of the Lagrange's multipliers $\chi = 0$ and $\boldsymbol{\chi} = 0$ the Lorentz gauge. The possibility of Lorentz gauge choice is connected with the idea of the gauge symmetry, the essence of which can be expressed in the following way: construct the field tensor from potentials in such a way that the differential constraints, which are a priori imposed on the currents and express physical laws of current conservation/interconversion, would be ***natural*** constraints for the base Lagrangian.

When using Lorentz gauge, the differential constraints (18) and (19) should not be included into the list of ***solvable*** problems under the numerical solution of field problems. They can be used as a tool for monitoring of accuracy and stability of the computation process.

3.3 Algebraic condition for the Singlet and Triplet Current Coupling

Existence of singlet current J^{ν} violates the isotropy of YM-space; it violates the equality of the three currents of YM-triplet $\mathbf{J}^{\nu}$, selecting from three YM-currents the one which the singlet current J^{ν} is closely coupled with. We shall denote this "selected" current by $\overset{3}{\mathrm{J}}{}^{\nu}$. Algebraic condition for S-current J^{ν} coupling with the selected third component of YM-triplet of currents $\overset{3}{\mathrm{J}}{}^{\nu}$ has the following form:

$$\overset{3}{\mathrm{J}}{}^{\nu}\left(\overset{3}{\mathrm{J}}_{\nu}-\mathrm{J}_{\nu}\right)=0. \tag{20}$$

The algebraic postulate (20) specifies the holonomic constraint, imposed on the currents of the singlet-triplet theory. It is based on the current structure of Weinberg-Salam electroweak theory ("WS theory", e.g. [5]). But the current structure of WS theory per se cannot be a "legal" basis for the ***classical*** theory that we are constructing. The currents in WS theory are algebraic expressions which are bilinear by the spinor wave functions of fermions. Point fermions are primary, non-interpretable objects of WS theory. They are built into the theory "manually". In WS theory, just as in the modern relativistic quantum theory in general, the totally ***quantum object*** – spinor wave function of a fermion – is implicitly interpreted as some true ***classical*** field under construction of the ***classical*** Lagrangian which afterwards, following the procedure of the secondary quantization (or Feinman integration by all possible configurations of all fields of the theory), transforms into a carrier of all the information about the quantum system. Such inconsistency, a latent "dragging" of the quantum object into the classical Lagrangian, dating back to the pioneer works of 1930th (V. Heisenberg, W. Pauli, V. Veiskopf), generates the condition of modern quantum relativistic physics, which was uncompromisingly described by a severe judge Paul Dirac as "an ugly and incomplete one" [6].

Here we are constructing the classical theory, in which the wave functions of particles can not appear in any way; and the massive particles are not built into the theory "manually". They are missing "at the entrance" of the theory, but they must appear as computable, constructed objects "at the exit" of the theory. "At the entrance" of the theory there are only currents and potentials, plus the base Lagrangian structure, as well as the holonomic and differential constraints which are imposed on the currents.

According to these remarks, the algebraic constraint, imposed on the currents of the theory (20), must be considered a postulate of the theory, without any apparent references to the existing WS theory. The accounting of this constraint leads to appearance in the additional Lagrangian of a term which we shall denote as $L_{ad(\psi)}$:

$$L_{ad(\psi)} = -\frac{1}{2p_T^2}\psi \overset{3}{\mathrm{J}}{}^{\nu} \left(\overset{3}{\mathrm{J}}_{\nu} - \mathrm{J}_{\nu}\right). \tag{21}$$

In formula (21), ψ is the Lagrange's multiplier, which is the functional of the space-time, coordinates x_μ. This very term in the Lagrangian "entangles" physics of the singlet maxwellian sector with physics of triplet Yang-Mills sector in case if current $\overset{3}{\mathrm{J}}{}^{\nu}$ is different from identical zero. The base Lagrangian (17) splits into singlet and triplet parts. The terms of the additional Lagrangian $L_{ad(\chi)}$ and $L_{ad(\boldsymbol{\chi})}$, generated by the differential constraints imposed on currents (if these terms are proper to calculate technically by numerical solution of a definite problem), are also placed according to the principle "each one in its own sector". And only the term (21) in the additional Lagrangian mixes two sectors of physics.

3.4 Algebraic "$2 \to 3$" Decomposition of the Coupled Pair of currents J^ν and $\overset{3}{\mathrm{J}}{}^{\nu}$

The algebraic condition (20), coupling S-current J^ν with the third YM-component of T-current $\mathbf{J}^\nu$, and the conditions for the space-likeness of currents (4), (5) make it possible to conduct an effective decomposition of a couple of currents J^ν, $\overset{3}{\mathrm{J}}{}^{\nu}$, by representing them in the form of a linear combination of the triple of mutually orthogonal currents N^ν, l^ν,r^ν, where current N^ν is isotropic and currents l^ν and r^ν are space-like:

$$\begin{aligned}
&\mathrm{J}^\nu = N^\nu + l^\nu + 2r^\nu,\\
&\overset{3}{\mathrm{J}}{}^{\nu} = l^\nu - N^\nu,\\
&N^\nu N_\nu = 0,\\
&l^\nu N_\nu = 0,\\
&r^\nu N_\nu = 0,\\
&l^\nu r_\nu = 0,\\
&l^\nu l_\nu < 0,\ r^\nu r_\nu < 0.
\end{aligned} \tag{22}$$

It is not difficult to see that the problem (22) has two one-parametric families of solutions. The condition for the existence of these solutions is the inequalities

$$\mathrm{J}^\nu \mathrm{J}_\nu < \overset{3}{\mathrm{J}}{}^{\nu} \overset{3}{\mathrm{J}}_{\nu} < 0. \tag{23}$$

It is proper to construct an explicit solution to the problem (22) within the "intrinsic frame of reference" of current $\overset{3}{\mathrm{J}}{}^{\nu}$, where time component of current $\overset{3}{\mathrm{J}}{}^{0}$ is missing and 3-vector of the current space components is oriented along the axisx^1:

$$\overset{3}{\mathrm{J}}{}^{\nu} = \{0,\, \alpha,\, 0,\, 0\}. \tag{24}$$

With regard to (24) and condition (20), the singlet current J^ν can be represented in this frame of reference in the following form:

$$\mathrm{J}^\nu = \{\tau,\, \alpha,\, \beta,\, 0\}\,; \quad \tau^2 < \beta^2. \tag{25}$$

In (25) it is established that through the rotation about axis x^1 we oriented axes x^2 and x^3 in a way that $\mathrm{J}^3 = 0$.
Inequality, connecting τ and β, results from the inequality (23).
Thus, r^ν, which at decomposition (22) enters only into the singlet current J^ν, is generally called the ***right-handed current***. Thus, l^ν, which enters into both S-current J^ν and the third component of YM-triplet $\overset{3}{\mathrm{J}}{}^\nu$, is called the ***left-handed current***. The isotropic current N^ν will be called a ***neutrino current***. The solution to the problem (22) for the neutrino current can have the following form:

$$N^\nu = N\,\left\{\beta,\, 0,\, \tau,\, \pm\sqrt{\beta^2-\tau^2}\right\}, \tag{26}$$

where N is an arbitrary normalization multiplier. It is proper to assume that $N = 1$. Fixation of parameter N in a specific frame of reference does not influence Lorentz-invariance of the solution: for transition to another frame of reference it is enough to make Lorentz-transformation of 4-vector (26). Under such gauge of N^ν, the left-handed and the right-handed currents have the following form:

$$\begin{aligned} l^\nu &= \left\{\beta,\, \alpha,\, \tau,\, \pm\sqrt{\beta^2-\tau^2}\right\}, \\ r^\nu &= \left\{\frac{\tau}{2}-\beta,\, 0,\, \frac{\beta}{2}-\tau,\, \mp\sqrt{\beta^2-\tau^2}\right\}. \end{aligned} \tag{27}$$

Currents l^ν and r^ν depend on the arbitrarily chosen gauge of parameter N, but their sum $j^\nu = l^\nu + r^\nu$ ("electromagnetic current") does not depend on the choice of N:

$$j^\nu = \left\{\frac{\tau}{2},\, \alpha,\, \frac{\beta}{2},\, 0\right\}.$$

Combination of currents q^ν also does not depend on the choice of normalization constant N:

$$q^\nu = r^\nu + N^\nu.$$

Electromagnetic current j^ν and current q^ν are reduced to the half-sum and, correspondingly, to the half-difference of currents J^ν and $\overset{3}{\mathrm{J}}{}^\nu$, and, consequently, they are as well-defined physical quantities as currents J^ν and $\overset{3}{\mathrm{J}}{}^\nu$. Currents N^ν, l^ν, r^ν themselves in the arbitrary singlet-triplet state are not well-defined physical quantities.
The procedure of "$2 \to 3$"-decomposition of currents, presented by formulas (26) and (27), does not only depend on the arbitrary gauge of parameter (and the choice of sign in z-components of vectors N^3, l^3, r^3) – this procedure is also not P-invariant. Indeed, the inversion of spatial axes means the reversal of sign of spatial components of any vector, including transformation $\alpha \to -\alpha$, $\beta \to -\beta$ in the components of currents J^ν

and $\overset{3}{\mathrm{J}}{}^{\nu}$ (formulas (24) and (25)). But, y-components of vectors N^2 and l^2, described by the formulas (26) and (27), do not change sign in the process, but time components N^0 and l^0 change the sign. Thus, under the inversion of spatial axes, all three currents N^ν, l^ν and r^ν turn into other physical entities. To restore the invariance of the theory relatively to discrete transformations, it is necessary to make T-transformation (change of the time coordinate sign) and C-transformation (change of sign of all currents and potentials) along with P-transformation (inversion of spatial coordinates). In so doing, to follow PCT-invariance of the theory in z-components of the currents (N^3, l^3, r^3) we have to transit to the second branch of the square root[5]. PCT-inversion leaves the procedure of "$2 \to 3$"-decomposition of currents invariant. This procedure itself is not necessary for the classical theory. Of course, there are such physical states, in which the terms "right-hand current", "left-handed current", "neutrino current" describe the observable physical quantities. The simplest of these states is a pure singlet state with a nondegenerate singlet current J ($\mathrm{J}^\nu \mathrm{J}_\nu < 0$) and an empty triplet sector ($\mathbf{J}^\nu = 0; \mathbf{W}^\nu = 0$). In this case we can assume that $l^\nu = 0$, $N^\nu = 0$, consequently, the singlet current J^ν is reduced to the right-hand current: $\mathrm{J}^\nu = 2r^\nu$. The extraction of a neutrino current is effective in special limit conditions when one or both inequality signs in formula (23) are replaced by equality signs. For example, if currents J^ν and $\overset{3}{\mathrm{J}}{}^{\nu}$ have identical nonzero pseudo-Euclidean modules:

$$\mathrm{J}^\nu \mathrm{J}_\nu = \overset{3}{\mathrm{J}}{}^{\nu} \overset{3}{\mathrm{J}}_{\nu} < 0, \tag{28}$$

and, accordingly, in formula (25) $\tau = \beta$, the system of equations (22) has no solution which would satisfy the condition of space-likeness of the right-hand current r^ν. The condition (28) describes "left-hand state":

$$\begin{aligned} r^\nu &= 0, \\ l^\nu &= \left\{ \frac{1}{2}\beta,\, \alpha,\, \frac{1}{2}\beta,\, 0 \right\}, \\ N^\nu &= \left\{ \frac{1}{2}\beta,\, 0,\, \frac{1}{2}\beta,\, 0 \right\}, \end{aligned}$$

and decomposition of currents has a simple form:

$$\begin{aligned} \mathrm{J}^\nu &= l^\nu + N^\nu, \\ \overset{3}{\mathrm{J}}{}^{\nu} &= l^\nu - N^\nu. \end{aligned} \tag{29}$$

In the left-hand singlet-triplet state (29) the left-hand current l^ν and neutrino current N^ν must be regarded as observable physical quantities. In the process, currents $\overset{1}{\mathrm{J}}{}^{\nu}$, $\overset{2}{\mathrm{J}}{}^{\nu}$ are sure to be present in the triplet sector.

One more special limit condition is a combined isotropization of both currents J^ν and $\overset{3}{\mathrm{J}}{}^{\nu}$:

$$\mathrm{J}^\nu \mathrm{J}_\nu = \overset{3}{\mathrm{J}}{}^{\nu} \overset{3}{\mathrm{J}}_{\nu} = 0.$$

[5] In language usually used in theoretical physics, this transition to the second branch of square root in the expression for neutrino current N^ν (26) can be interpreted as transition from neutrino to antineutrino.

In this condition, in formulas (23) we should suppose that $l^\nu = 0$ and $r^\nu = 0$, and, consequently, according to (22), we should accept that:

$$\begin{aligned} \mathrm{J}^\nu &= N^\nu, \\ \overset{3}{\mathrm{J}}{}^\nu &= -N^\nu. \end{aligned} \tag{30}$$

In such mixed singlet-triplet state, the same neutrino current N^ν is present both in the singlet and triplet sector ("Maxwell-Yang-Mills neutrino").
There are two more special one-sector neutrino states.
The first one is the singlet neutrino state with "empty" triplet sector:

$$\begin{aligned} &\mathrm{J}^\nu = N^\nu; \quad N^\nu N_\nu = 0 \\ &\mathbf{J}^\nu = 0; \quad \mathbf{W}^\nu = 0. \end{aligned}$$

In this state the decomposition formula (22) "does not work" – there is no triplet partner for current J^ν.
This one-sector singlet neutrino state ("maxwellian neutrino") is studied in detail in the article [1] of this series[6]. Algebraic condition (20), imposed on the currents, will be also satisfied at the "empty" singlet sector in case if current $\overset{3}{\mathrm{J}}{}^\nu$ is a neutrino one:

$$\begin{aligned} &\mathrm{J}^\nu = 0; \quad \mathrm{W}^\nu = 0. \\ &\overset{3}{\mathrm{J}}{}^\nu = N^\nu; \quad N^\nu N_\nu = 0. \end{aligned} \tag{31}$$

In this case the decomposition formulas (22) "do not work": current $\overset{3}{\mathrm{J}}{}^\nu$ does not have a singlet partner.
Relations (31) describe a one-sector triplet neutrino state (one of the variants of "Yang-Mills neutrino").

3.5 Algebraic Condition for the Inner Normalization of Yang-Mills Triplet

The current structure of the standard version of electroweak WS-theory with point particles indicates the existence of one more algebraic constraint, imposed on the three components of YM-triplet:

$$2\,\overset{1}{\mathrm{J}}{}^\nu\,\overset{2}{\mathrm{J}}_\nu = \overset{3}{\mathrm{J}}{}^\nu\,\overset{3}{\mathrm{J}}_\nu\,. \tag{32}$$

While we take the condition (32) as a postulate of the triplet theory, it registers some kind of "auto-normalization" of the currents inside YM-triplet. This condition creates its own contribution $L_{ad(\eta)}$ to the additional Lagrangian L_{ad}:

$$L_{ad(\eta)} = \frac{\eta}{2p_T^2}\left(2\,\overset{1}{\mathrm{J}}{}^\nu\,\overset{2}{\mathrm{J}}_\nu - \overset{3}{\mathrm{J}}{}^\nu\,\overset{3}{\mathrm{J}}_\nu\right), \tag{33}$$

[6] The description of this state in [1] contained reservations expressing a doubt as to reality of such neutrino state: the principal text of the article [1] was already written in the late 1970s, and it took the author a long period of time to put up with such Maxwellian model of neutrino.

in which η is a Lagrange's multiplier which depends on the space-time coordinates x^ν. This contribution to the Lagrangian is totally placed in YM-sector of the theory.
The very fact of appearing of the algebraic constraints (20) and (32) in the theory, which are beyond the base Lagrangian of the theory, is rather unpleasant from the point of view of an estimation of the completeness and correctness of our ideas of fundamentals of physics. The Lagrange's multipliers ψ and η, entering into the additional Lagrangian, actually correspond to some outer "Higgs-like" fields, which influence the real physical fields. In the language of Newtonian mechanics they correspond to the "constraints force", imposed on the current fields by ***someone*** from outside. The theory could be rehabilitated by constructing such solutions for the theory equations, for which algebraic constraints (20) and (32) are ***natural***, i.e. are satisfied under the absence of "Higgs-like" fields ψ and η: $\psi \equiv 0$ and $\eta \equiv 0$.

3.6 About Quality Criteria of the Classical Physical Theory

We can assume that a good classical physical theory has to satisfy two of the following quality criteria:

- the theory contains a short , not requiring any extension, list of primary objects of the theory which form the base Lagrangian;
- the theory does not contain any external to the Lagrangian constraints imposed on the primary objects, or these constraints are ***natural*** for the Lagrangian of the theory and do not generate any external "Higgs-like" constraint forces, at least for some quite wide range of solutions.

The existing version of theoretical physics – both Weinberg-Salam electroweak theory and the Standard Model – actually does not satisfy the first criterion. Point fermions with their wave functions are built into the theory "manually" as primary objects. There can be as many fundamental fermions as possible: only experimental data restricts the length of the list of primary objects of the theory. The existing version of physics also does not satisfy the second criterion, as it contains external Higgs objects.
The version of classical field theory, developed here, satisfies the first criterion. In this version the short list of primary objects of the theory is a priori fixed: they are currents only the singlet and triplet ones (and octuplet, if strong interactions are taken into account). There is nothing in nature except them and dynamically conjugated with them potentials with the same Yang-Mills dimension (singlet, triplet and octuplet). Particles and waves are secondary, derivative, calculated entities; they are the objects "at the exit" of the theory – there can be as many of them as possible.
We do not know if the theory, developed here, satisfies the second criterion. We have no theorem which would contain the proof of the fact that there exists quite a wide range of solutions to the equations of singlet-triplet theory, such as that algebraic constraints (20) and (32), imposed on the currents, are natural for the base Lagrangian of the theory

and can be satisfied under zero values of "Higgs-like" fields ψ and η[7]. These solutions describe all singlet-triplet states, which are realized in nature.
Two more criteria have to be added to the two quality criteria of the fundamental physical theory, formulated above.

- The theory naturally incorporates the accounting of Riemann space-time curvature.
- The theory must contain the solutions with finite energy and should not require renormalization or regularization.

The theory, developed here, satisfies these criteria, while Weinberg-Salam theory and the Standard Model do not.
And, finally, a good physical theory has to satisfy the fifth, Einsteinian, quality criterion:

- The theory should not contain arbitrarily specified dimensionless "empirical" constants. The fine structure constant, the mixing angles, the mass ratio of particles, etc. must be calculated within the theory[8].

There is no theory satisfying this Einsteinian criterion in modern physics. The theory, developed here, partly satisfies this criterion, apparently, allowing calculating mass ratio of the particles.

4. Discrete Transformations in the Singlet-Triplet Theory

4.1 Charge Conjugation in the Singlet-Triplet Theory

The field theory, constructed here, – and its base Lagrangian L (17) and additional Lagrangian L_{ad}, consisting of the sum of $L_{ad}(\psi)$ (21) and $L_{ad}(\eta)$ (33), - are invariant relative to the charge conjugation consisting of two operations:
(1) Sign reversal for all currents and potentials;
(2) Permutation of Yang-Mills indices $1 \leftrightarrow 2$.
Formally it can be written with help of two sector operators of the charge conjugation, operator $\hat{C}_S$, affecting the singlet vectors and reduced to change of their sign:

$$\mathrm{J}_\nu \rightarrow \mathrm{J}'_\nu = \hat{C}_S \mathrm{J}_\nu = -\mathrm{J}_\nu,$$
$$\mathrm{W}_\nu \rightarrow \mathrm{W}'_\nu = \hat{C}_S \mathrm{W}_\nu = -\mathrm{W}_\nu,$$

and operator $\hat{C}_T$, affecting YM-triplets of vectors. Operator $\hat{C}_T$ is YM-matrix, changing vector signs and permutating YM-indices $1 \leftrightarrow 2$:

$$\mathbf{J}_\nu \rightarrow \mathbf{J}'_\nu = \hat{C}_T \mathbf{J}_\nu,$$
$$\mathbf{W}_\nu \rightarrow \mathbf{W}'_\nu = \hat{C}_T \mathbf{W}_\nu,$$

[7] We also do not know if there exists at least one mathematician who is able to get interested in this problem.
[8] "God had no choice", – this is probably the answer that must be given by good physical theory to the famous Albert Einsteins question.

where $\hat{C}_T = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

The structure of singlet-triplet theory itself demonstrates "inequality" of Lorentz vectors entering YM-triplet. The third YM-component of current $\overset{3}{\mathrm{J}}_\nu$ is connected with singlet current J_ν by relation (20) and is thereby distinguished in Yang-Mills triple. It is obvious that this extraction has to be also transferred to the third YM-component of potential $\overset{3}{\mathrm{W}}_\nu$. "Inequality" of YM-indices 1 and 2 ("chiral inequality") is connected with the availability of the vector YM-product in determining of the triplet field tensor $\mathbf{W}_{\mu\nu}$ (15) and differential constraint (19). In these expressions vector YM-product is summed up with YM-vector. For this summation operation to make invariant geometric sense in YM-space, the vector YM-product should not be an axial YM-vector, but a true YM-vector. Therefore, with the specified third YM-vectors component we must absolutely definitely choose the numbering scheme of the first and the second components. We should suppose that we have a possibility to provide a chiral consistency, i.e. to choose the right-handed (or the left-handed) coordinate system simultaneously and consistently in the coordinate space and YM-vector space[9]. To provide chiral consistency at the inversion of a sign of YM-currents and YM-potentials, we have to permutate YM-indices $1 \leftrightarrow 2$.

At the performing of the described transformation of charge conjugation, field tensors are transformed in the same way as the currents and potentials:

$$\mathrm{W}_{\mu\nu} \to \mathrm{W}'_{\mu\nu} = \hat{C}_S \mathrm{W}_{\mu\nu},$$
$$\mathbf{W}_{\mu\nu} \to \mathbf{W}'_{\mu\nu} = \hat{C}_T \mathbf{W}_{\mu\nu}.$$

The base Lagrangian, quadratic by currents, potentials and field tensors, does not change with charge conjugation.

To conclude paragraph 4.1, we will dare to make two anticipatory remarks. Probably, these remarks will help the reader to estimate the defiant beauty and complexity of Yang-Mills physics.

(1) In octuplet sector of physics, which is responsible for strong interactions and is the subject for discussion in the subsequent article of this series, there is no octuplet matrix of charge conjugation similar to triplet matrix $\hat{C}_T$ by its characteristics. In other words, in contrast to the singlet and triplet sector, the octuplet sector of physics is charge-asymmetrical. No permutation of vector numbers in Yang-Mills octuplet provides invariance of the octuplet Lagrangian relative to the inversion of currents.

No doubt, this fact is not psychologically easy to put up with. Perhaps, it is as hard to do as it was for the physicists of 1956 to put up with the parity violation. However, the charge asymmetry, latent in the octuplet algebra itself, obviously makes

[9] "Only a theory defines what can be observable", – Albert Einstein once said to Werner Heinsenberg.

it possible to understand the reason of the evident and defiant charge asymmetry of the Universe: in the octuplet sector God ***had a choice***.

(2) The requirement to observe chiral definiteness of the solutions of Yang-Mills equations actually imposes some unilateral (releasing) constraint on variables of the triplet and octuplet sectors of physics. Such constraint is not something unusual for the problems of classical mechanics, for example, the constraint between two material points, established by means of a nonstretchable line. Problems of mechanics with such unilateral constraint lead to solutions, discontinuous by the velocity vector: at the moments of "constraint activation" some velocity components change a sign for the opposite. If you do not like solutions with velocity discontinuity, you are free to complicate the constraint model for example, to consider line stretchability.

However, in the problems of mathematical physics and the field theory, unilateral constraints, apparently, did not occur before. Besides, unlike a mechanical problem with nonstretchable line, we do not have any more accurate and adequate model in reserve: there is nothing beyond Yang-Mills physics at all, this is ***all*** physics.

In particular, considering the problems of Yang-Mills free no-current waves, we easily find out that Yang-Mills nonlinear wave equations possess chiral symmetry, despite the chiral asymmetry of the field tensor and the Lagrangian. Accordingly, the solutions of these equations possess chiral symmetry. Here we encounter not a very pleasant dilemma:

a) to take beautiful, continuous and smooth chiral-symmetric solutions which contain the most difficult and unpredictable chaotic oscillations and to shut our eyes to the requirement of chiral determinacy of the solution;

or

b) to take into account the requirement of chiral determinacy and "to impose" chiral determinacy upon the solution by means of rough "chiralization" procedure; the solutions become discontinuous by derivatives, lose their chaotic character and gain a rather boring periodicity[10].

Choosing the first alternative leads to good Yang-Mills ***mathematical*** physics, but to the knowingly incorrect Yang-Mills ***physics*** which violates the requirement of chiral determinacy of solutions.

The choice of the second alternative correctly reproduces physical requirements of Yang-Mills chiral definiteness, but generates rather ugly version of Yang-Mills mathematics of solutions with derivatives discontinuities.

The very necessity for discussion of this "mathematics-physical" dilemma seems strange after half-a-century existence of Yang-Mills equations. Apparently, it is connected with the fact that during these fifty years there has been made no serious attempt to turn Yang-Mills classical field theory into the similar minutely developed section of mathematical physics and applied mathematics, which is classical electrodynamics, for example.

[10] In the subsequent articles of this series the examples of numerical solutions to the problems of Yang-Mills triplet and octuplet waves both chaotic chiral-symmetric solutions and periodic solutions generated by the procedure of chiralization will be provided.

4.2 Combined Coordinate Inversion in the Singlet-Triplet Theory

When making the operation of inversion of coordinate system:

$$x_\nu \to x'_\nu = -x_\nu, \tag{34}$$

it should be taken into account that occurring transformation of the three-dimensional right-handed coordinate system to the left-handed one (or vice versa) requires the same transformation in YM-space, i.e. permutation of YM-indices $1 \leftrightarrow 2$. However, during the performance of the transformation (34) and YM-permutation, different terms in the expression for YM-tensor of field $\mathbf{W}_{\mu\nu}$ (15) behave differently. In the coordinate inversion, Lorentz vectors of currents and potentials reverse signs, consequently, the expressions in the form of $\partial_\nu \mathbf{J}^\nu$ or $\partial_\nu \mathbf{W}_\mu$ do not reverse a sign. However, vector products in the form of $\mathbf{W}_\mu \times \mathbf{J}^\mu$ or $\mathbf{W}_\mu \times \mathbf{W}_\nu$ do reverse a sign due to permutation of YM-indices $1 \leftrightarrow 2$.
One more operation has to be provided for restoration of the consistency of transformational qualities of different terms in the same formula – sign inversion of currents and potentials (without repeated permutation of the indices $1 \leftrightarrow 2$). As a result, both the base Lagrangian and additional Lagrangian of the singlet-triplet theory prove to be invariant relative to this three-way combined coordinate inversion:
(1) Inversion of signs x_ν, generating the inversion of signs J_ν, W_ν, $\mathbf{J}_\nu$ and $\mathbf{W}_\nu$.
(2) Permutation of YM-indices $1 \leftrightarrow 2$.
(3) Repeated inversion of the signs of currents and potentials.
Finally, when performing such combined inversion, currents and potentials do not change, field tensors reverse a sign (with regard to permutation of YM-indices $1 \leftrightarrow 2$); the Lagrangian does not change.
Coordinate inversion (34) per se, not accompanied by charge inversion, is not a correctly determined procedure in the triplet sector of physics[11].

5. About the Name of the Theory

The standard name of this branch of physics, going back to Abdul Salam, is "electroweak theory". This name is absolutely justified from the historical point of view: unification within the framework of one theory of electromagnetic and weak interaction was a great achievement at the time of WS theory creation. However, from the point of view of logical structure of this theory, we believe that it is possible to leave the terms "electromagnetic interaction" and "weak interaction" in the past and to speak about "singlet-triplet theory" (ST theory) or, at least, about "Maxwell and Yang-Mills theory" (MYM theory).

[11] The triviality of this statement within the framework of Yang-Mills structure of physics deserves real amazement. This statement is equivalent to so-called "Law of parity non-conservation in weak interactions". The formulation of this "law" in due time required some courage from T. D. Lee and C. N. Yang. Probably, Yang, partly under the influence of Wolfgang Pauli' unfair criticism, who was always confident of his own innocence, did not treat Yang-Mills equations, named after him, with due trust.

In fact, besides the idea of C.N. Yang and R.L. Mills about the existence of triplet of YM- currents and triplet of YM potentials [3] conjugated with it, the idea of Sheldon Lee Glashow about the necessity of ***joint*** interpretation of Maxwell singlet and Yang-Mills triplet [7] was also needed, so, from the point of view of personification of the theory it would be, perhaps, better to speak about "Maxwell-YangMills-Glashow theory". It was S. Glashow's work, in which angle , later called "Weinberg angle", appeared for the first time. Glashow's work, performed six years after the publication of C.N. Yang and R.L. Mills' article [3], did not contain any mentioning of the article [3] at all, so to a certain extent it is possible to say that anyway, the electroweak theory would have probably been constructed without C.N. Yang and R.L. Mills' contribution. However, Yang-Mills' approach goes beyond the framework of the triplet sector of physics without any slightest changes in its essence and extends to the octuplet strong interaction. The only technical change necessary here is the replacement of Levi-Civita three-index three-dimensional symbol (which is a structural symbol of the group SU(2)) in the definition of vector product of YM-vectors by a three-index eight-dimensional structural symbol $\overset{abc}{f}$ of the group SU(3). (Over-letter YM indices in the octuplet sector of physics run values from one to eight). Therefore, personification of the name of the theory in this case seems doubtful, so the best way to name the theory is by its basic algebraic structure: the singlet-triplet theory (ST-theory) or, with regard to the octuplet sector of physics, a "singlet-triplet-octuplet theory" (STO-theory).

The commonly acknowledged version of electroweak theory existing today, Weinberg-Salam theory, has also added Higgs sector to the singlet and triplet sectors of physics. Within the framework of the classical field approach, developed in this article, and the previous article of this series [1], the introduction to the theory of Higgs' fields seems needless[12]: the theory does not require Higgs' spontaneous symmetry violation for generation of particles' own weight; masses of particles are directly present in the ST theory as its computable parameters "at the exit" of the theory.

6. The Effective Lagrangian of the Singlet-Triplet Theory

By coupling the base Lagrangian of the ST- theory (17) with the additional Lagrangian, which accounts the constraints, imposed on the variables of ST- theory, and accepting Lorentz gauge of the theory (i.e. omitting terms $L_{ad(\chi)}$ and $L_{ad(\boldsymbol{\chi})}$), we obtain the effective

[12] If Steven Weinberg asked me why Higgs' field is missing from this theory, just like emperor Napoleon asked Pierre Simon de Laplace about the reason of the Creator's absence in Laplace's "Mecanique celeste", I would respond with Laplace's words: "Sire, je n'ai pas en besoin de cette hypothese" ("I had no need of that hypothesis, Your Majesty") [8]. But Higgs' fields were inseparable part of theoretical physics of the recent decades. It is hard to imagine that physicists would easily refuse to use Higgs language. "Laplace's Escapade has never been what belongs to the masses", – Pierre Chaunu [9] once said.

Lagrangian of the ST- theory:

$$L' = L'_S + L'_T + L'_{ST}. \qquad (35)$$

In this formula (35), L'_S is a part of the effective Lagrangian which contains only the variables belonging to the singlet sector; L'_T contains only the quantities belonging to the triplet sector and L'_{ST} is a part of the Lagrangian responsible for mixing of singlet and triplet sectors:

$$\begin{aligned}
L'_S &= -\frac{1}{8p_S^2}\mathrm{J}^\nu \mathrm{J}_\nu - \frac{1}{2p_S}\mathrm{J}^\nu \mathrm{W}_\nu - \frac{1}{16\pi}\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu};\\
L'_T &= -\frac{1}{8p_T^2}\mathbf{J}^\nu \mathbf{J}_\nu - \frac{1}{2p_T}\mathbf{J}^\nu \mathbf{W}_\nu - -\frac{1}{16\pi}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{\eta}{2p_T^2}\left(2\,\overset{1}{\mathrm{J}}{}^\nu\,\overset{2}{\mathrm{J}}_\nu - \overset{3}{\mathrm{J}}{}^\nu\,\overset{3}{\mathrm{J}}_\nu\right); \qquad (36)\\
L'_{ST} &= -\frac{\psi}{2p_T^2}\,\overset{3}{\mathrm{J}}{}^\nu\left(\overset{3}{\mathrm{J}}_\nu - \mathrm{J}_\nu\right).
\end{aligned}$$

The effective Lagrangian (35), (36) (we will call it "the four-current Lagrangian") is appropriate for description of many physical states which differ in number of currents involved into the current state, from zero-current states (a free singlet field or a free triplet field) to total four-current states. The four-current Lagrangian has to be changed in case of isotropization of at least one of the four currents in some 4-zone with non-zero 4-volume. In this case the contribution of this current in the current part of the Lagrangian (36) is missing, but the term of the form $\lambda N^\nu N_\nu$ appears in the additional Lagrangian, where N^ν is isotropic (neutrino) current, and λ is the Lagrange's multiplier. Let us note that retention of η and ψ in the Lagrangian is equivalent to introduction into the theory of some external "Higgs-like" fields that provide realization of the constraints laid onto currents. For constructing a closed physical theory, it would be reasonable to accept that the corresponding current constraints are natural for the singlet-triplet theory Lagrangian, i.e. constraint equations can be satisfied at zero values of the Lagrange's multipliers η and ψ. We have no official proof of the validity of this assumption.
The condition of current isotropy for neutrino states is unnatural, and the term of the form $\lambda N^\nu N_\nu$ cannot be omitted from the Lagrangian in the process of field equations obtaining. It means that the neutrino state cannot cover all the four-dimensional space-time while occupying only some finite zone, which is transitional between other states.

7. Classification of States of the Singlet-Triplet Theory

All variety of the states, described by the Lagrangian of the ST-theory (36), can be classified as follows. First of all it is possible to allocate the "pure" or "one-sector" states which completely fit one of the two sectors of the theory: singlet states (S-states or M-states) and triplet states (T-states or YM-states). The states which are not pure will be named "mixed" or "two-sector" states (ST-states). By means of rescaling of variables it is possible to except Weinberg angle from the description of pure states. Weinberg angle

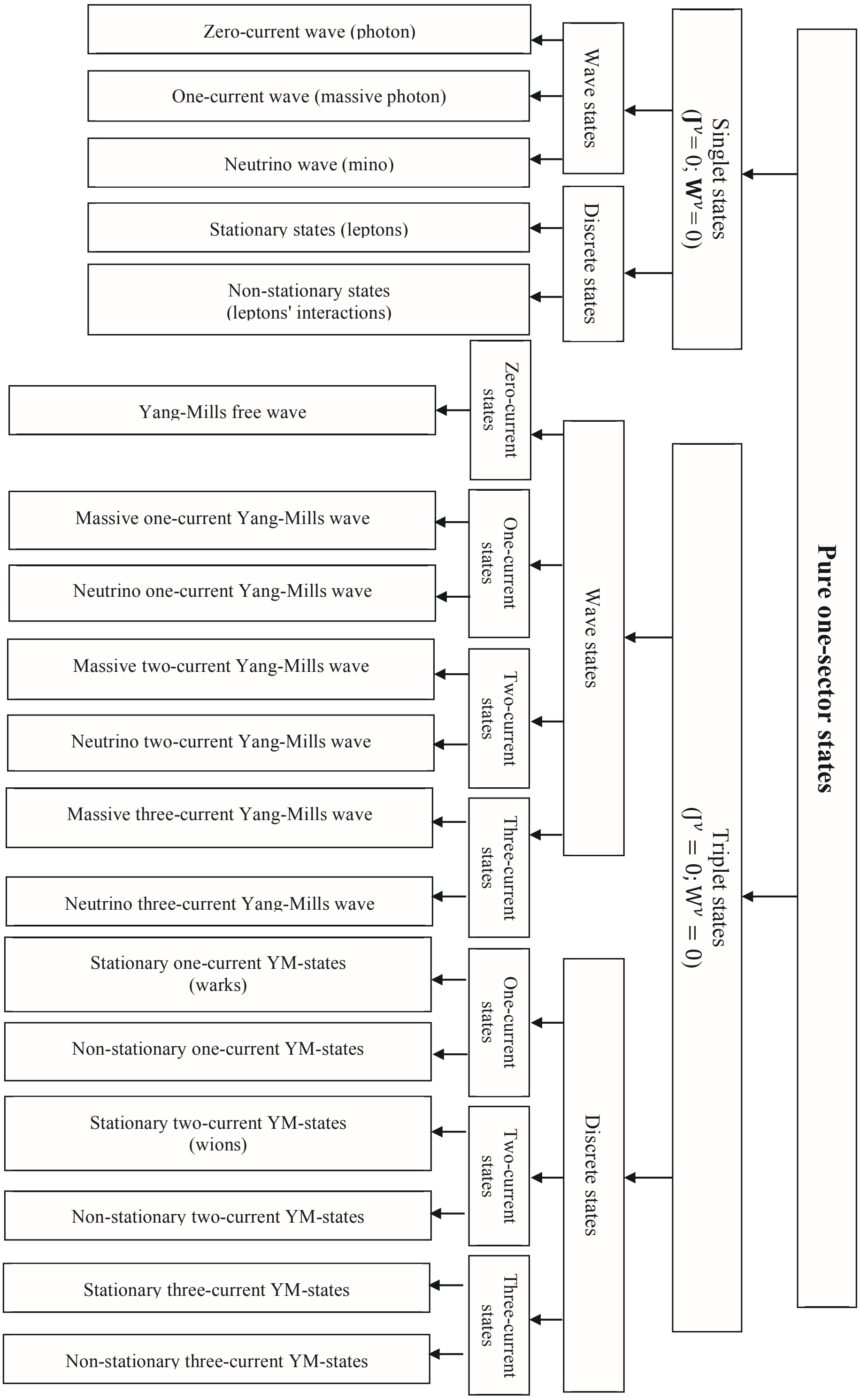

Fig. 1 Pure one-sector states in the *ST*-theory

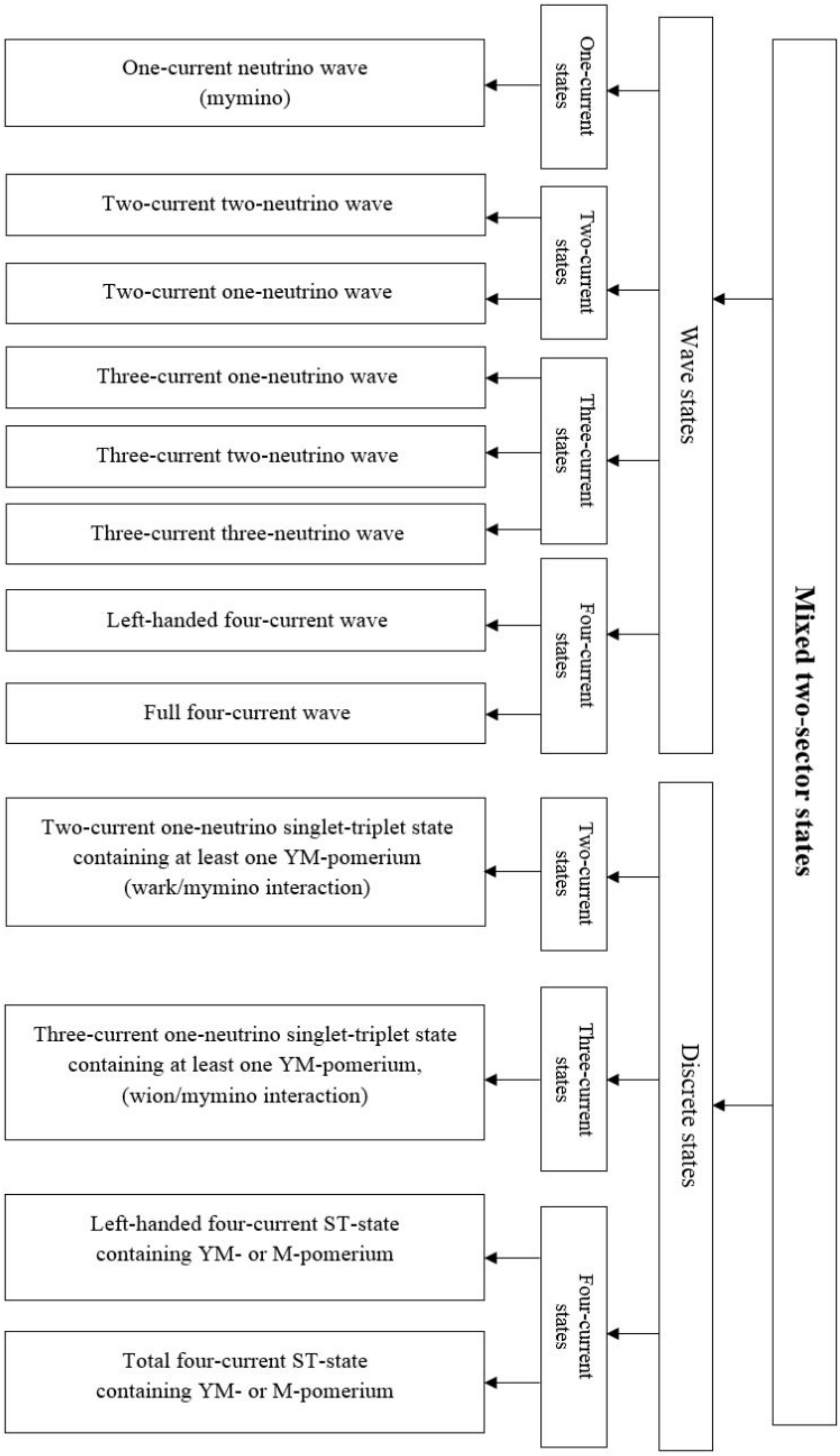

Fig. 2 Mixed two-sector states in the *ST*-theory

cannot be excepted from the description of mixed states. Further the states of the ST-theory can be classified by the number of nonzero currents: zero-current states, i.e. free fields are maxwellian free field in pure S-state and Yang-Mills free field in pure T-state; one-current states, two-current, three-current and four-current states. Singlet states can be only zero-current or one-current. All states, except zero-current ones, can be classified by number of pomeriums, i.e. outer boundaries of the current zones. "Wave" states have no pomerium at all (the current zone is unbounded). Photon, and also massive photon and maxwellian neutrino, described in article [1] of this series, relate to the wave states in the singlet sector. In the triplet sector there are wave states which are similar to them. The states which have at least one pomerium will be named "discrete states". The most important of the discrete states is a "stationary state", i.e. a state with one pomerium for which there is such an intrinsic frame of reference where currents and fields do not depend on time. Stationary states describe current structure of massive single particles. Stationary states exist only as pure one-sector states, mixed stationary states do not exist.

It is possible to distinguish a special case of "neutrino states" among current wave states – i.e. the states in which pseudo-Euclidean square of 4-current vector is everywhere equal to zero.

There are mixed neutrino states simultaneously fitting two sectors of the theory, Maxwellian and Yang-Mills' ("Maxwell-Yang-Mills neutrino", or ***mymino*** in abbreviated form). There is a pure singlet neutrino state (makswellian neutrino, or ***mino*** in abbreviated form). There are pure neutrino states which completely fit Yang-Mills sector ("Yang-Mills neutrino", or ***ymino*** in abbreviated form). We do not know whether this theoretical classification of neutrino states – ***mino***, ***ymino***, ***mymino*** – corresponds to the empirical fact of distinction of the lepton neutrinos (electronic neutrino, muonic neutrino, τ-neutrino)[13].

The described classification of states, possible in the ST-theory, is reflected in fig. 1 (pure states) and fig. 2 (mixed states). These pictures illustrate a variety of particular problems, which can be formulated and solved within the framework of the ST-theory, and thereby, the inner wealth of the ST-theory as a section of mathematical and computer physics.

By describing formulations of the problems of the ST-theory further in the text of this article, we restrict ourselves to only two types of the problems: the wave problems containing no pomerium at all and stationary problems with one pomerium.

Wave problems are quite correct as problems of classical mathematical physics, however the search for correspondence between a variety of their solutions and real quantum objects of the micro-world may appear a difficult task[14].

[13] For complete description of neutrino states it can be noted that a priori nothing forbids to allow the possibility of izotropization of any of eight currents of the octuplet sector of physics which is not considered in this article, and thereby to enter into consideration one more set of neutrinos different from ***mino***, ***ymino*** and ***mymino***.

[14] Within the framework of the wave problems, for example, it is not difficult to consider the theory of standing Yang-Mills waves – triplet and octuplet (gluon) ones. But how, by means of what equipment,

Stationary classical single-particle problems allow to calculate, in principle, masses, charges and spins of particles and to study the inner structure of massive particles. The particles of one physical class consisting of one current, differ in topology of the outer boundaries of pomerium, and also in number and topology of the inner boundaries of latens. Particles of different physical classes differ in currents they consist of. Within the framework of this article we are not going to consider the formulations of the problems which describe the states with more than one pomerium. Within classical physics they are obviously non-stationary states. These states are the key ones for modern treatment of physics of micro-world. They are the states of particles interaction, including the states corresponding to spontaneous decay of unstable particles. The possibility of correct description of such states within the framework of the classical ST-theory, constructed here, seems rather doubtful. However, without studying such classical problems, it is difficult to count on successful construction of the proper quantum theory of non-point particles. The following restrictions on the variables, entering into the Lagrangian of the singlet-triplet theory (36), are used for extraction of the particular states reflected in figures 1 and 2.

A. Wave states

A1. One-sector wave states

- Zero-current singlet wave (free maxwellian wave, photon):
  $\mathbf{J}^\nu \equiv 0; \quad \mathbf{W}^\nu \equiv 0; \quad \mathrm{J}^\nu \equiv 0.$
- One-current singlet wave (heavy photon):
  $\mathbf{J}^\nu \equiv 0; \quad \mathbf{W}^\nu \equiv 0; \quad \mathrm{J}^\nu \mathrm{J}_\nu < 0.$
- Neutrino singlet wave (maxwellian neutrino, myno):
  $\mathbf{J}^\nu \equiv 0; \quad \mathbf{W}^\nu \equiv 0; \quad \mathrm{J}^\nu \neq 0; \quad \mathrm{J}^\nu \mathrm{J}_\nu = 0.$
- Zero-current triplet wave (Yang-Mills free wave):
  $\mathrm{J}^\nu \equiv 0; \quad \mathrm{W}^\nu \equiv 0; \quad \mathbf{J}^\nu \equiv 0.$
- One-current triplet wave (Yang-Mills heavy one-current wave):
  $\mathrm{J}^\nu \equiv 0; \quad \mathrm{W}^\nu \equiv 0; \quad \overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}{}_\nu < 0; \quad \overset{2}{\mathrm{J}}{}^\nu \equiv 0; \quad \overset{3}{\mathrm{J}}{}^\nu \equiv 0.$[15]
- Neutrino one-current triplet wave (Yang-Mills neutrino, ymino):
  $\mathrm{J}^\nu \equiv 0; \quad \mathrm{W}^\nu \equiv 0; \quad \overset{1}{\mathrm{J}}{}^\nu \neq 0; \quad \overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}{}_\nu = 0; \quad \overset{2}{\mathrm{J}}{}^\nu = 0; \quad \overset{3}{\mathrm{J}}{}^\nu = 0.$[16]
- Two-current triplet wave (Yang-Mills heavy two-current wave):
  $\mathrm{J}^\nu \equiv 0; \quad \mathrm{W}^\nu \equiv 0; \quad \overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}{}_\nu = 0; \quad \overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}{}_\nu < 0; \quad \overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}{}_\nu < 0; \quad \overset{3}{\mathrm{J}}{}^\nu \equiv 0.$
- Neutrino two-current triplet wave:
  $\mathrm{J}^\nu = 0; \mathrm{W}^\nu = 0; \overset{1}{\mathrm{J}}{}^\nu \neq 0; \overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}{}_\nu < 0; \overset{2}{\mathrm{J}}{}^\nu \neq 0; \overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}{}_\nu = 0; \overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}{}_\nu = 0; \overset{3}{\mathrm{J}}{}^\nu \equiv 0.$
- Three-current triplet wave (Yang-Mills heavy three-current wave):

would it be possible to conduct Yang-Mills analogue of maxwellian experiment on observation of standing singlet waves which was conducted by O. Wiener for the first time already in 1890th? For observing standing gluon waves it is necessary to arrange reflecting surfaces in a baryon. And, even if this arrangement is possible at baryon collisions, the characteristic dimensions of atomic nuclei are enormously high in comparison with the fundamental length r_0, appearing in the theory of currents and potentials.

[15] Also two analogous wave states with arbitrary permutation of YM-indices.

[16] Also two analogous neutrino states with arbitrary permutation of YM-indices.

$\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu < 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu < 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu < 0$; $2\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = \overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu$.

- Neutrino three-current triplet wave:
  $\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \neq 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu = 0$; $\overset{2}{\mathrm{J}}{}^\nu \neq 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = 0$; $\overset{3}{\mathrm{J}}{}^\nu \neq 0$; $\overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu = 0$.

A2. Two-sector wave states

- One-current two-sector neutrino wave (Maxwell-Yang-Mills neutrino, mymino):
  $\mathrm{J}^\nu = N^\nu$; $\overset{3}{\mathrm{J}}{}^\nu = -N^\nu$; $N^\nu N_\nu = 0$; $\overset{1}{\mathrm{J}}{}^\nu \equiv 0$; $\overset{2}{\mathrm{J}}{}^\nu \equiv 0$.
- Two-current two-sector one-neutrino wave:
  $\mathrm{J}^\nu = N^\nu$; $\overset{3}{\mathrm{J}}{}^\nu = -N^\nu$; $N^\nu N_\nu = 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu < 0$; $\overset{2}{\mathrm{J}}{}^\nu \equiv 0$.[17]
- Two-current two-sector two-neutrino wave:
  $\mathrm{J}^\nu = N^\nu$; $\overset{3}{\mathrm{J}}{}^\nu = -N^\nu$; $N^\nu N_\nu = 0$; $\overset{1}{\mathrm{J}}{}^\nu \neq 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu = 0$; $\overset{2}{\mathrm{J}}{}^\nu \equiv 0$.[18]
- Left-handed four-current wave:
  $\mathrm{J}^\nu = l^\nu + N^\nu$; $\overset{3}{\mathrm{J}}{}^\nu = l^\nu - N^\nu$; $N^\nu \neq 0$; $N^\nu N_\nu = 0$; $l^\nu N_\nu = 0$; $l^\nu l_\nu < 0$;
  $2\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = l^\nu l_\nu$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu < 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu < 0$.
- Full four-current wave:
  $\mathrm{J}^\nu \mathrm{J}_\nu < 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu < 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu < 0$; $\overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu < 0$; $2\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = \overset{2}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu < 0$; $\overset{3}{\mathrm{J}}{}^\nu \left(\overset{3}{\mathrm{J}}_\nu - \mathrm{J}_\nu\right) = 0$.

B. Discrete states

B1. One-sector discrete states

- Singlet stationary state (lepton):
  $\mathrm{J}^\nu \mathrm{J}_\nu \leq 0$; $\mathbf{J}^\nu \equiv 0$; $\mathbf{W}^\nu \equiv 0$; there is only one pomerium; there is intrinsic reference system of a state in which $\dfrac{\partial}{\partial t} \equiv 0$.
- Singlet non-stationary state (electromagnetic interaction of leptons):
  $\mathrm{J}^\nu \mathrm{J}_\nu \leq 0$; $\mathbf{J}^\nu \equiv 0$; $\mathbf{W}^\nu \equiv 0$; there is more than one pomerium.
- Stationary one-current triplet state (hypothetical particle wark):
  $\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{2}{\mathrm{J}}{}^\nu \equiv 0$; $\overset{3}{\mathrm{J}}{}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; there is only one YM-pomerium; there is intrinsic reference system of state in which $\dfrac{\partial}{\partial t} \equiv 0$.[19]
- Non-stationary one-current triplet state (YM-interaction of warks):
  $\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{2}{\mathrm{J}}{}^\nu \equiv 0$; $\overset{3}{\mathrm{J}}{}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; there are more than one YM-pomeriums[20].
- Stationary two-current triplet state(hypothetical particle wion):
  $\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{3}{\mathrm{J}}{}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu \leq 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = 0$; there is one YM-pomerium, common for currents $\overset{1}{\mathrm{J}}{}^\nu$ and $\overset{2}{\mathrm{J}}{}^\nu$; there is intrinsic state reference system in which $\dfrac{\partial}{\partial t} \equiv 0$.
- Non-stationary two-current triplet state (YM-interaction of warks or/and wions):
  $\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu \leq 0$; $\overset{3}{\mathrm{J}}{}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = 0$; there is more than one

[17] Also analogous one-neutrino wave with permutation of YM-indices $1 \leftrightarrow 2$.
[18] Also analogous two-neutrino wave with permutation of YM-indices $1 \leftrightarrow 2$.
[19] Also analogous stationary state with permutation of YM-indices $1 \leftrightarrow 2$.
[20] Also analogous non-stationary state with permutation of YM-indices $1 \leftrightarrow 2$.

YM-pomerium.

- Stationary three-current triplet state:
  $\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu \leq 0$; $\overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu \leq 0$; $2\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = \overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu$; there is one YM-pomerium which is common for the three YM-currents; there is intrinsic reference system of state in which $\frac{\partial}{\partial t} \equiv 0$.
- Non-stationary three-current triplet state:
  $\mathrm{J}^\nu \equiv 0$; $\mathrm{W}^\nu \equiv 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu \leq 0$; $\overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu \leq 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = \overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu$; there is more than one pomerium.

B2. Two-sector discrete states

- Two-current one-neutrino singlet-triplet state containing at least one YM-pomerium (wark/mymino interaction):
  $\mathrm{J}^\nu = N^\nu$; $\overset{3}{\mathrm{J}}{}^\nu = -N^\nu$; $N^\nu N_\nu = 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{2}{\mathrm{J}}{}^\nu = 0$.[21]
- Three-current one-neutrino singlet-triplet state containing at least one YM-pomerium, common for currents $\overset{1}{\mathrm{J}}{}^\nu$ and $\overset{2}{\mathrm{J}}{}^\nu$ (wion/mymino interaction):
  $\mathrm{J}^\nu = N^\nu$; $\overset{3}{\mathrm{J}}{}^\nu = -N^\nu$; $N^\nu N_\nu = 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu \leq 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = 0$.
- Left-handed four-current singlet-triplet state containing at least one pomerium:
  $\mathrm{J}^\nu = l^\nu + N^\nu$; $\overset{3}{\mathrm{J}}{}^\nu = l^\nu - N^\nu$; $l^\nu N_\nu = 0$; $N^\nu N_\nu = 0$; $l^\nu l_\nu \leq 0$;
  $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu \leq 0$; $2\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = l^\nu l_\nu$.
- Total four-current singlet-triplet state containing at least one pomerium:
  $\mathrm{J}^\nu \mathrm{J}_\nu \leq 0$; $\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu \leq 0$; $\overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu \leq 0$; $2\overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = \overset{3}{\mathrm{J}}{}^\nu \overset{3}{\mathrm{J}}_\nu$; $\overset{3}{\mathrm{J}}{}^\nu \left(\overset{3}{\mathrm{J}}_\nu - \mathrm{J}_\nu\right) = 0$.

The provided classification of the states of singlet-triplet theory is rather formal. It describes possible states on the basis of a set of non-zero currents. Each state, allocated in the classification, needs some existence theorem which would claim that the algebraic conditions, imposed on currents, are compatible[22].

Wave states can be researched quite minutely, at least, for the plane waves. Plane waves can be described by a system of ordinary differential equations. The solutions to the problems of plane waves within the framework of the singlet-triplet theory can be obtained analytically or numerically.

Discrete states in the singlet-triplet theory are described by an equation system in partial derivatives which include a priori unknown boundaries of current zones (pomerium and latens). The lack of existence theorems causes some difficulties for the development of numerical methods of solving problems of discrete states.

The given classification does not include transitional neutrino states, i.e. such states, in which 4-zone, occupied with the neutrino current, is finite and non-stationary.

The given classification for some discrete states is not unambiguously definite. For example, the state, described as a "singlet non-stationary state", describes both the state of

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[21] Also analogous one-neutrino state with permutation of YM-indices $1 \leftrightarrow 2$.

[22] But stationary states also need some spectral theorem claiming ***non-uniqueness*** of stationary state with the given set of algebraic conditions for currents.

scattering of a lepton on a lepton, for which the ***classical*** theory, constructed here, can be quite correct, and the state of annihilation of two leptons, for which, probably, the classical theory can prove to be as unacceptable as Newtonian mechanics for description of atoms.

However, we believe that ***classical*** singlet-triplet theory is worth careful and detailed development as a section of mathematical physics for clarification of prognostic opportunities of the theory and limits of its applicability.

8. Pure one-sector states: Maxwellian Singlet States

8.1 Getting Rid of Weinberg Angle: the Electromagnetic Lagrangian

In pure S-state all triplet currents $\mathbf{J}^\nu$ and all triplet potentials $\mathbf{W}^\nu$ are equal to zero. Only L'_S will remain in the effective ST-Lagrangian (36). Supposing that in L'_S:

$$\begin{aligned} &\underset{em}{\mathrm{j}}^{\nu} = \frac{1}{2}\mathrm{J}^\nu; \\ &\mathrm{A}^\nu = p_S \mathrm{W}^\nu; \\ &\mathrm{A}^{\mu\nu} = \partial^\mu \mathrm{A}^\nu - \partial^\nu \mathrm{A}^\mu = p_S \mathrm{W}^{\mu\nu}, \end{aligned} \tag{37}$$

we can rewrite the singlet Lagrangian L'_S in the following form:

$$L'_S = \frac{1}{p_S^2} \underset{em}{L},$$

where

$$\underset{em}{L} = -\frac{1}{2} \underset{em}{\mathrm{j}}^{\nu} \underset{em}{\mathrm{j}}_{\nu} - \underset{em}{\mathrm{j}}^{\nu} \mathrm{A}_\nu - \frac{1}{16\pi} \mathrm{A}_{\mu\nu} \mathrm{A}^{\mu\nu}. \tag{38}$$

Therefore, the singlet Lagrangian L'_S differs from the standard Lagrangian of classical electrodynamics of non-point particles [1] only in insignificant common multiplier.

The electromagnetic Lagrangian (38) comprises the description of both the wave problems without current boundaries (an ordinary no-current electromagnetic wave – photon; "heavy" current electromagnetic wave – "heavy photon"; neutrino wave – maxwellian neutrino [1]), and the discrete problems containing outer and inner boundaries of current zones (pomerium and latens). The discrete problems, as a subset, comprise stationary singlet problems which solution describes inner structure of massive leptons [1].

"Deductibility" of the electromagnetic Lagrangian (38) from the effective ST-Lagrangian (36) in one-sector maxwellian problem is an absolutely trivial mathematical fact. But this fact comprises the profound physical meaning: ***intrinsic characteristics of massive leptons are formed in the singlet sector of the theory and do not depend on the triplet interactions***. In particular, the formulas of heavy lepton mass ratios, provided in the article [1], remain reasonable in the ST-theory. The triplet sector of the theory influences the stability of stationary states of the singlet sector, but does not

influence the stationary states themselves.
Now we do not know whether the correct formulation of the problem on stability of the singlet stationary discrete state within the framework of the ***classical*** theory, considered here, is possible. Perhaps, the effective ST-Lagrangian (36) comprises the description of such instability connected with spontaneous "swelling" of the bubble of the neutrino triplet currents and this bubble's "splashing" beyond the limits of pomerium of a discrete singlet object.

8.2 Estimate of the Fundamental Constant r_0

The classical theory of non-point particles, constructed here, contains as "entrance" parameters of the theory the fundamental length r_0, the velocity of light c, the fundamental constant j_0 which determines the top limit of the current density, and the dimensionless constant k which forms Einstein equations and is proportional to the constant of gravity G [1].
If we had a numerical solution of the stationary one-sector singlet problem, we would have a possibility to express the intrinsic characteristics of massive leptons – mass m, charge e and angular momentum M – through these "entrance" parameters. But now we have neither such numerical solution, nor even the existence theorem which would guarantee the existence of at least one solution; nor the spectral theorem which would guarantee the existence of some finite or countable set of the intrinsic solutions, differing in topological characteristics. In such situation everything we can afford is dimensional estimates.
The mass of a lepton can be estimated by the order of magnitude by means of the following formula:

$$mc^2 \approx \frac{r_0^2}{c^2}\,\mathrm{j}_0^2\, r_P^2\, r_0,$$

and the electrical charge e – by means of the formula:

$$e \approx \frac{1}{c}\mathrm{j}_0\, r_0^3.$$

These formulas are based on the following ideas. The main contribution to the total energy of the massive lepton is made by a small part of the lepton volume in small neighborhood of the "cavitated" tube latebra, on the boundary of which the current density achieves a peak value j_0. We believe that the diametrical dimension of cavitated tubes is of the order of Planckian length r_P, and the typical diametrical dimension of the current high density zone, bordering upon latens, is approximately the same. The longitudinal dimension of this zone is of the order of the general outer dimension of the lepton, and this dimension, in its turn, is close to the fundamental constant r_0. The first dimensional factor $\left(r_0^2/c^2\right)$ in the formula for the lepton mass is the proportionality coefficient which forms the expression of the tensor components of energy-momentum the current field (see [1]).

The expression for charge dimension e, linear by current density, means that the noticeable contribution to the total value of an electric charge is made not only by a narrow zone in the neighborhood of "cavitated" tube latebra, but by the whole lepton volume as well. The assumptions, on which basis the above mass and charge estimates are constructed, are open to criticism. But let us see the consequences these estimates lead to. From the given estimates for m and e we find out that

$$\frac{e^2}{mc^2} \approx \frac{r_0^3}{r_P^2},$$

but

$$\frac{e^2}{mc^2} = r_T,$$

where r_T is a classical Thomson lepton radius.
So, on the grounds of these lepton mass and charge estimates, we get a possibility to estimate the unknown fundamental constant r_0:

$$r_0 \approx \left(r_T r_P^2\right)^{1/3}. \tag{39}$$

If we use Thomson electron radius $r_T \approx 2.8 \cdot 10^{-13}$ cm as a quantity r_T, (39) gives the following estimate for r_0:

$$r_0 \approx 0.9 \cdot 10^{-26}\text{cm}. \tag{40}$$

The given estimate (40) is probably not too cogent. Formula (39) contains not only the fundamental Planckian length r_P, but also Thomson radius of a specific lepton; the dimension r_0 itself has to be fundamental, i.e. to have no "binding" to any specific particle. However, we have no other estimate of the fundamental constant r_0.
Quantity r_0 (40) is enormously small from the point of view of the possibility ever "to consider" the electron structure in any feasible physical experiment. However, this quantity is quite large in comparison with Planckian length r_P determining the diametrical dimensions of latebrae. Physics of leptons contains two dimensionless constants: the constant of fine structure α and ratio ${}^{r_P}\!/_{r_0}$. Undoubtedly, these two constants have to be somehow connected with each other.
By estimating, within the framework of the same "naive estimate", the own angular momentum of lepton M by the formulas provided in the article [1], it is possible to find that

$$\mathrm{M} \approx \frac{r_0^2}{c^2} \cdot \frac{1}{c} \cdot \mathrm{j}_0^2 \cdot r_0^4.$$

Determining ratio $\dfrac{e^2}{\mathrm{M}c}$ by means of this estimate, we get the estimate which is not very substantial:

$$\frac{e^2}{\mathrm{M}c} = O(1), \tag{41}$$

at the same time, as the correct estimate could be $\mathrm{M} \approx \hbar$ and $\dfrac{e^2}{\mathrm{M}c} \approx \alpha$, where $\hbar$ is Planckian constant and α is a fine structure constant. We do not know now whether estimate (41) is the evidence of a rough and not removable imperfection of the constructed theory, or whether this estimate can be improved under the accurate numerical solution of a stationary discrete singlet problem and accurate calculation of quantities e and M.

9. Pure One-sector States: Yang-Mills Triplet States

9.1 Getting Rid of Weinberg Angle: the Weak Lagrangian

The set of pure YM-states, represented in fig.1 have an "empty" singlet sector: $\mathrm{J}^{\nu} = 0$; $\mathrm{W}^{\nu} = 0$. The missing of the singlet current, in accordance with the algebraic condition (20), means that

$$\overset{3}{\mathrm{J}}{}^{\nu}\,\overset{3}{\mathrm{J}}_{\nu} = 0. \tag{42}$$

Condition (42) means that current $\overset{3}{\mathrm{J}}{}^{\nu}$, if it is not identically zero, can be only a neutrino one. Algebraic condition of the inner normalization of currents of YM-triplet (32) requires, in this case, the orthogonality of the first and second components of YM-triplet.

$$\overset{1}{\mathrm{J}}{}^{\nu}\,\overset{2}{\mathrm{J}}_{\nu} = 0. \tag{43}$$

Weinberg angle can be eliminated from the triplet part of the effective ST-Lagrangian by means of scaling which comprises the triplet interaction coefficient p_T:

$$\begin{aligned} &p_T \mathbf{W}^{\nu} = \mathbf{A}^{\nu};\\ &\mathbf{A}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + \mathbf{A}_\mu \times \mathbf{A}_\nu = p_T \mathbf{W}_{\mu\nu}, \end{aligned} \tag{44}$$

Let us also re-scale the triplet currents, assuming that:

$$\overset{1}{\mathrm{j}}{}^{\nu} = \frac{1}{2}\,\overset{1}{\mathrm{J}}{}^{\nu};\ \overset{2}{\mathrm{j}}{}^{\nu} = \frac{1}{2}\,\overset{2}{\mathrm{J}}{}^{\nu};\ N^{\nu} = \frac{1}{2}\,\overset{3}{\mathrm{J}}{}^{\nu}, \tag{45}$$

and, in accordance with (42) and (43):

$$\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}_{\nu} = 0, \tag{46}$$

$$N^{\nu} N_{\nu} = 0. \tag{47}$$

After substitution of the relations (44) and (45), with regard to (46) and (47), the triplet Lagrangian L'_T (36) can be formulated in the following form:

$$L'_T = \frac{1}{p_T^2} L_w, \tag{48}$$

$$\begin{aligned} L_w = &-\frac{1}{2}\left(\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{1}{\mathrm{j}}_{\nu} + \overset{2}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}_{\nu}\right) - \left(\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{1}{\mathrm{A}}_{\nu} + \overset{2}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{A}}_{\nu} + N^{\nu}\,\overset{3}{\mathrm{A}}_{\nu}\right)\\ &- \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot\mathbf{A}_{\mu\nu} - \eta\,\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}_{\nu} - \frac{1}{2}\lambda N^{\nu} N_{\nu}. \end{aligned} \tag{49}$$

The Lagrangian L_w does not include Weinberg angle θ_w.
Let us name the Lagrangian L_w "weak Lagrangian", the triplet of currents $\left(\overset{1}{\mathrm{j}}{}^{\nu},\, \overset{2}{\mathrm{j}}{}^{\nu},\, N^{\nu}\right)$, which satisfies the conditions (46) and (47), – "weak currents triplet", triplet $\mathbf{A}^{\nu}$ – "weak potentials triplet", and tensor $\mathbf{A}^{\mu\nu}$ – "weak field tensor". Coefficients η and λ under corresponding terms in the weak Lagrangian are Lagrange's multipliers.

9.2 Zero-current Triplet States: Yang-Mills Free Triplet Waves

9.2.1 Yang-Mills Wave Equation

All the currents of YM-triplet are missing in the wave triplet state:

$$\overset{1}{\mathrm{j}}{}^{\nu} = \overset{2}{\mathrm{j}}{}^{\nu} = N^{\nu} = 0.$$

The Lagrangian of such zero-current state is reduced to Yang-Mills free field Lagrangian $L_{w,f}$:

$$L_{w,f} = -\frac{1}{16\pi}\,\mathbf{A}_{\mu\nu}\cdot\mathbf{A}^{\mu\nu}. \tag{50}$$

Variation of action functional with the Lagrangian (50) by weak potentials $\mathbf{A}_{\mu}$ gives the equations of the free weak field:

$$\partial_{\mu}\mathbf{A}^{\mu\nu} + \mathbf{A}_{\mu}\times\mathbf{A}^{\mu\nu} = 0. \tag{51}$$

By eliminating field tensor $\mathbf{A}^{\mu\nu}$ from Yang-Mills field equations (51) by means of (44), we can present the equations (51) in the following form:

$$-\Box\mathbf{A}^{\nu} + \mathbf{A}_{\mu}\times\left(2\partial^{\mu}\mathbf{A}^{\nu} - \partial^{\nu}\mathbf{A}^{\mu} + \mathbf{A}_{\mu}\times\mathbf{A}^{\nu}\right) = \partial^{\nu}\ell + \mathbf{A}^{\nu}\times\ell. \tag{52}$$

In equations (52) ℓ is 4-divergence of the weak potential $\mathbf{A}^{\mu}$:

$$\ell = \partial_{\mu}\mathbf{A}^{\mu},$$

and $\Box$ is D'Alembert operator:

$$\Box = -\partial_{\mu}\partial^{\mu}.$$

One arbitrary gauge condition can be imposed on potentials $\mathbf{A}^{\mu}$. It is proper to assume that 4-divergence ℓ is identically zero. This equation is compatible with the field equations (52). Finally, the free weak field of the zero-current triplet state is described by the following system of the equations:

$$\Box\mathbf{A}^{\nu} = \mathbf{A}_{\mu}\times\left(2\partial^{\mu}\mathbf{A}^{\nu} - \partial^{\nu}\mathbf{A}^{\nu} + \mathbf{A}_{\mu}\times\mathbf{A}^{\nu}\right); \tag{53}$$

$$\partial_{\mu}\mathbf{A}^{\mu} = 0. \tag{54}$$

9.2.2 Yang-Mills Plane Zero-current Wave

The simplest solution to field equations (53), (54) can be tried in the form of a plane wave with wave vector k^ν, supposing that 12 unknown functions of $\mathbf{A}^\mu$ depend only on a single argument – a wave phase ϕ:

$$\phi = k_\mu x^\mu.$$

For the plane wave, the field equations (53), (54) transform into a system of ordinary differential equations:

$$k_\mu k^\mu \mathbf{A}^{\nu\prime\prime} + 2k_\mu \mathbf{A}^\mu \times \mathbf{A}^{\nu\prime} - k^\nu \mathbf{A}_\mu \times \mathbf{A}^{\mu\prime} + \mathbf{A}_\mu \times (\mathbf{A}^\mu \times \mathbf{A}^\nu) = 0. \tag{55}$$

$$k_\mu \mathbf{A}^{\mu\prime} = 0. \tag{56}$$

In these equations the prime next to the potentials stands for the derivative with respect to wave phase ϕ.

The plane wave equations (55) have apparent scale invariance with respect to wave vector k^μ.

Let k^μ be a time-like vector:

$$k_\mu k^\mu = \varepsilon^2 > 0.$$[23]

Let us make scale transformation of potentials $\mathbf{A}^\mu$, coordinates x^μ and wave vector k^μ, supposing that

$$k^\nu = \varepsilon \tilde{k}^\nu; \quad \tilde{k}^\nu \tilde{k}_\nu = 1.$$

$$\mathbf{A}^\nu = \varepsilon \tilde{\mathbf{A}}^\nu; \quad x^\nu = \frac{1}{\varepsilon}\tilde{x}^\nu.$$

At this transformation the wave phase remains invariable:

$$\phi = k^\nu x_\nu = \tilde{k}^\nu \tilde{x}_\nu.$$

The specified scale transformation turns a wave vector into a unit vector without changing the form of the wave equations (55), (56). Let us rewrite these equations with new variables, omitting the symbol $\sim$ over the corresponding quantities in the form:

$$\mathbf{A}^{\nu\prime\prime} + 2k_\mu \mathbf{A}^\mu \times \mathbf{A}^{\nu\prime} - k^\nu \mathbf{A}^\mu \times {\mathbf{A}_\mu}' + \mathbf{A}_\mu \times (\mathbf{A}^\mu \times \mathbf{A}^\nu) = 0; \tag{57}$$

[23] The fact that both W. Pauli and C.N. Yang with R.L. Mills in 1953-1954 passed by the classical Yang-Mills wave theory with the time-like wave vector, can be considered an inexplicable historical paradox. With persistence, that is worth regrets, there was considered only the version of the theory with isotropic wave vector $k^\mu k_\mu = 0$, generating the idea of zero mass of Yang-Mills field quantum ("one will always obtain vector mesons with rest mass zero", wrote Wolfgang Pauli in 1953 December [4]) under general quantum treatment.
It is obvious that the construction of the solution (55) does not require any wave vector isotropy at all. This sight aberration of the triplet theory authors is probably caused by the tendency of the physicists of that time to treat nonlinear equations by means of perturbation methods, which is absolutely unacceptable for Yang-Mills *essentially nonlinear* equations.
Finally, this stable aberration led to the introduction of Higgs mechanism for explanation of particles masses. The possibility of the correct treatment of Yang-Mills waves with time-like wave vector, described in this article, shows the existence of a rest mass of such waves, but within the framework of the classical theory, this mass, proportional to the pseudo-Euclidean module of the wave 4-vector, remains arbitrary.

$$k_\mu \mathbf{A}^{\mu\prime} = 0; \tag{58}$$

$$k^\mu k_\mu = 1. \tag{59}$$

From the condition (58) follows that convolution $k_\mu \mathbf{A}^\mu$ does not depend on the wave phase:

$$k_\mu \mathbf{A}^\mu = \mathbf{q} = \mathrm{const};$$

which allows to rewrite the plane YM-wave equation in the form:

$$\mathbf{A}^{\nu\prime\prime} + 2\mathbf{q} \times \mathbf{A}^\nu - k^\nu \mathbf{A}_\mu \times \mathbf{A}^{\mu\prime} + \mathbf{A}_\mu \times (\mathbf{A}^\mu \times \mathbf{A}^\nu) = 0. \tag{60}$$

9.2.3 Energy Integral for Yang-Mills Plane Zero-current Wave

By multiplying the wave equation (60) scalarly by the derivative of potential $\mathbf{A}'_\nu$, and integrating by the wave phase ϕ, it is possible to obtain the first integral of the wave equation (60) which is appropriate to be called "energy integral":

$$\begin{aligned} &\mathfrak{K} + \mathfrak{U} = \mathrm{E} = \mathrm{const}; \\ &\mathfrak{K} = -\frac{1}{2}\,\mathbf{A}^{\nu\prime} \cdot \mathbf{A}_\nu{}'; \\ &\mathfrak{U} = \frac{1}{4}\left((\mathbf{A}^\nu \cdot \mathbf{A}_\nu)^2 - (\mathbf{A}_\mu \cdot \mathbf{A}_\nu)(\mathbf{A}^\mu \cdot \mathbf{A}^\nu)\right). \end{aligned} \tag{61}$$

It is easy to show that physical quantity $\mathfrak{U}$, which can be treated as a potential wave energy density, is positive-definite. Physical quantity$\mathfrak{K}$ is the first term in the energy conservation law (61) which can be interpreted as the density of kinetic wave energy; it is also positive-defined. Therefore, wave energy is positive.
In wave equation (60) and energy integral (61) it is easy to see that Yang-Mills plane wave has a self-similarity relative to the energy quantity E.
The scale transformation which uses E as a parameter:

$$\begin{aligned} &\mathbf{A}^\nu = \mathrm{E}^{1/4}\,\overset{*}{\mathbf{A}}{}^\nu, \\ &\phi = -\mathrm{E}^{1/4}\,\overset{*}{\phi}, \\ &\mathbf{q} = \mathrm{E}^{1/4}\,\overset{*}{\mathbf{q}}, \end{aligned} \tag{62}$$

normalizes the wave energy to unit value without altering the form of the wave equation (60) and energy integral (61).
We will afford once again to rewrite almost tautologically the problem formulation of Yang-Mills plane wave in variables $\overset{*}{\mathbf{A}}{}^\nu$, $\overset{*}{\phi}$, $\overset{*}{q}$ introduced by the scale transformation (62) omitting symbol * over the letters:

$$\mathbf{A}^{\nu\prime\prime} + 2\mathbf{q} \times \mathbf{A}^{\nu\prime} - k^\nu \mathbf{A}^\mu \times \mathbf{A}_\mu{}' + \mathbf{A}_\mu (\mathbf{A}^\mu \cdot \mathbf{A}^\nu) - \mathbf{A}^\nu (\mathbf{A}_\mu \cdot \mathbf{A}^\mu) = 0, \tag{63}$$

$$-\frac{1}{2}\mathbf{A}^{\nu\prime} \cdot \mathbf{A}_\nu{}' + \frac{1}{4}\left((\mathbf{A}^\nu \cdot \mathbf{A}_\nu)^2 - (\mathbf{A}_\mu \cdot \mathbf{A}_\nu)(\mathbf{A}^\mu \cdot \mathbf{A}^\nu)\right) = 1, \tag{64}$$

$$k_\mu \mathbf{A}^\mu = \mathbf{q}, \tag{65}$$

$$k^{\mu}k_{\mu} = 1. \tag{66}$$

Let us pay attention to the fact that the invariance of Yang-Mills plane wave theory, relative to scale transformations generated by pseudo-Euclidean module of the wave vector and energy (or wave amplitude), means that Yang-Mills wave theory has neither geometrical optics approximation, nor asymptotics of small amplitudes (low energies). Both wave vector and wave energy can always be re-scaled to unit values.

9.2.4 Nonholonomic Constraint Equations for Yang-Mills Plane Zero-current Wave

By multiplying the wave equation (63) scalarly by wave vector k_{ν}, taking into account (65) and (66), we will get the identity which has to be satisfied with the solution (63):

$$\mathbf{A}^{\mu} \times \mathbf{Q}_{\mu} = 0. \tag{67}$$

where

$$\mathbf{Q}_{\mu} = \mathbf{A}'_{\mu} + \mathbf{A}_{\mu} \times \mathbf{q}. \tag{68}$$

The equation (67) with vector $\mathbf{Q}_{\mu}$ (68) can be treated as a non-holonomic constraint imposed on Yang-Mills dynamic system (63). It is sufficient to require the initial data to meet the condition (67).

9.2.5 Yang-Mills Plane Zero-current Wave in the Intrinsic Frame of Reference

It is proper to make further analysis of Yang-Mills plane wave in the intrinsic frame of reference of the wave, in which the wave vector k^{μ} has the form:

$$k^{\mu} = \{1;\, 0;\, 0;\, 0\}. \tag{69}$$

In this frame of reference the wave phase coincides with the intrinsic time of the frame, and the phase derivatives coincide with the intrinsic time derivatives. We shall denote these derivatives with a dot as a dot upper index after Lorentz indices. In this frame none of the potential components depend on spatial coordinates: the wave "flashes" and "dies out" simultaneously over the whole infinite three-dimensional space.

Our deep-rooted Lorentz intuition, undoubtedly, protests against this picture, however there is no violation of Lorentz-invariance: Yang-Mills plane wave with a fixed wave vector k^{ν} is the artificial mathematical entity which fills up the whole three-dimensional space. Such object requires the infinite stock of energy for its formation. The real wave, covering the finite spatial domain and bearing the finite total energy, is a wave packet, i.e. superposition of waves with different wave vectors. Such wave can be presented in the form of Fourier integral on four-dimensional space of the wave vectors. The more compact the space-time localization of the wave packet is, the wider is the spread of waves within the space of wave vectors. The notion "intrinsic frame of reference" makes sense only for a plane wave with a fixed wave vector. For a wave packet this notion has no sense. However, the mathematics of Yang-Mills wave packets, which requires Fourier transformation of Yang-Mills wave equations (63), seems to us almost "uninterpretable" in

the continual situation. The problem of Yang-Mills stationary waves, requiring a discrete Fourier transformation of the wave equation (63), is quite "interpreted". However, we do not know whether the engineering-physical problem of creation of "resonators" of Yang-Mills waves, which are analogous to the radio engineering resonators of electromagnetic waves, is solvable in principle.
Let us enter ordinary "1+3" notations for the components of the weak potential $\mathbf{A}^\nu$:

$$\overset{a}{\mathrm{A}}{}^{\nu} = \left\{ \overset{a}{\mathrm{T}}; \overset{a}{\mathbf{U}} \right\}, \tag{70}$$

where $\overset{a}{\mathrm{T}}$ is a time component of YM-potential vector a, $\overset{a}{\mathbf{U}}$ is the three spatial components of YM-potential vector a. Vectors $\overset{a}{\mathbf{U}}$ (for each a) are ordinary three-dimensional vectors in Euclidean three-dimensional coordinate space. The separation of the components of 4-vector into time and spatial part is not Lorentz - invariant. Let us assume that separation (70) is carried out in YM-wave intrinsic frame of reference. Transition to any other frame of reference is carried out by means of Lorentz- transformation of 4-vector $\overset{a}{\mathrm{A}}{}^{\nu}$.
Substituting a "separation formula" (70) into the condition (65) and taking into account the mode of the wave vector in the wave intrinsic frame of reference (69), we find that all the time components of YM-potentials are constant:

$$\overset{a}{\mathrm{T}} = \overset{a}{\mathrm{q}} = \text{const}.$$

Further we will assume that all $\overset{a}{\mathrm{q}}$ are zero:

$$\overset{a}{\mathrm{q}} = 0. \tag{71}$$

A non-zero value of constants $\overset{a}{\mathrm{q}}$ would mean the presence of some outer stationary field, which is hardly compatible with the problem of studying Yang-Mills ***free wave***.
By substituting (70) and (71) into the wave equation (63) and nonholonomic constraint (67), (68), we can represent the formulation of the problem of Yang-Mills plane wave in the following form:

$$\overset{a}{\mathbf{U}}{}^{\bullet\bullet} + \overset{b}{\mathbf{U}} \times \left(\overset{a}{\mathbf{U}} \times \overset{b}{\mathbf{U}} \right) = 0; \tag{72}$$

$$\overset{abc}{\varepsilon} \overset{b}{\mathbf{U}} \cdot \overset{c}{\mathbf{U}}{}^{\bullet} = 0. \tag{73}$$

In these formulas $\overset{a}{\mathbf{U}}$ is the triple of ordinary three-dimensional vectors, the symbol "×" means the ordinary three-dimensional vector product; a, b, c are Yang-Mills indices taking the values 1, 2, 3; $\overset{abc}{\varepsilon}$ is a three-dimensional Levi-Civita symbol; summation is made by YM-indices. Opening the triple vector product in (72), Yang-Mills equation (72) can be rewritten as follows:

$$\overset{a}{\mathbf{U}}{}^{\bullet\bullet} + \overset{ab}{\mathrm{S}} \overset{b}{\mathbf{U}} - \overset{bb}{\mathrm{S}} \overset{a}{\mathbf{U}} = 0. \tag{74}$$

Yang-Mills matrix $\overset{ab}{\mathrm{S}}$ appears in the relation (74):

$$\overset{ab}{\mathrm{S}} = \overset{a}{\mathrm{A}}{}^{\nu} \overset{b}{\mathrm{A}}_{\nu} = \overset{a}{\mathrm{T}} \overset{b}{\mathrm{T}} - \overset{a}{\mathbf{U}} \cdot \overset{b}{\mathbf{U}}, \tag{75}$$

or, regarding the condition (71) in the wave intrinsic system:

$$\overset{ab}{\mathrm{S}} = -\overset{a}{\mathbf{U}} \cdot \overset{b}{\mathbf{U}} . \tag{76}$$

A more compact notation (74) is formed if Yang-Mills "inertia tensor" $\overset{ab}{\mathrm{I}}$ is introduced:

$$\overset{ab}{\mathrm{I}} = \overset{ab}{\mathrm{S}} - \overset{ab}{\delta}\overset{cc}{\mathrm{S}}, \tag{77}$$

where $\overset{ab}{\delta}$ is Kronecker symbol in the space of YM-indices. In the intrinsic wave system, Yang-Mills inertia tensor $\overset{ab}{\mathrm{I}}$ with (76) looks as follows:

$$\overset{ab}{\mathrm{I}} = \mathrm{U}^2 \overset{ab}{\delta} - \overset{a}{\mathbf{U}} \cdot \overset{b}{\mathbf{U}}, \tag{78}$$

where $\mathrm{U}^2 = \sum_a \overset{a}{\mathbf{U}}{}^2$. By using tensor $\overset{ab}{\mathrm{I}}$, introduced here, Yang-Mills equations (74) can be rewritten in the form which remotely resembles the form of Euler equations in the dynamics of a perfectly rigid body:

$$\overset{a}{\mathbf{U}}{}^{\bullet\bullet} + \overset{ab}{\mathrm{I}}\overset{b}{\mathbf{U}} = 0. \tag{79}$$

Yang-Mills equations (79) (which would probably be appropriate to be called Yang-Mills-Euler ones) describe a three-vector dynamical system with nine degrees of freedom, where three nonholonomic constraints are imposed on this system's behavior (73). Let us name this dynamical system "tripletic Yang-Mills' oscillator" (tymos). During quantization of triplet Yang-Mills wave[24] there must appear a particle with a nonzero rest mass. By analogy with a photon – the singlet field quantum – this particle can be named ***ymon***. There are no reasons for identification of ***ymon*** with W- and Z-particles of Weiberg-Salam theory.

In notations (70) and (75) energy integral for ***tymos*** (79) has the following form:

$$\frac{1}{2}\overset{a}{\mathbf{U}}{}^{\bullet} \cdot \overset{a}{\mathbf{U}}{}^{\bullet} + \frac{1}{4}\left(\left(\overset{ab}{\mathrm{S}}\right)^2 - \overset{ab}{\mathrm{S}}\overset{ab}{\mathrm{S}}\right) = 1, \tag{80}$$

or, using formula (76) for YM-matrix $\overset{ab}{\mathrm{S}}$:

$$\frac{1}{2}\overset{a}{\mathbf{U}}{}^{\bullet} \cdot \overset{a}{\mathbf{U}}{}^{\bullet} + \frac{1}{2}\left(\left(\overset{1}{\mathbf{U}} \times \overset{2}{\mathbf{U}}\right)^2 + \left(\overset{2}{\mathbf{U}} \times \overset{3}{\mathbf{U}}\right)^2 + \left(\overset{3}{\mathbf{U}} \times \overset{1}{\mathbf{U}}\right)^2\right) = 1. \tag{81}$$

[24] In the article on the classical theory of Yang-Mills fields it is irrelevant to discuss these rules of quantization: it is sufficient to assume that they exist.

9.2.6 The Detailed Notation of the Tymos Dynamic Equations

Let us present here the detailed notation of Yang-Mills-Euler equations (79) with explicitly written Yang-Mills' and vector indices in Cartesian coordinates:

$$
\begin{aligned}
&\overset{1}{U}_1{}^{\bullet\bullet} + \overset{11}{I}\,\overset{1}{U}_1 + \overset{12}{I}\,\overset{2}{U}_1 + \overset{13}{I}\,\overset{3}{U}_1 = 0,\\
&\overset{1}{U}_2{}^{\bullet\bullet} + \overset{11}{I}\,\overset{1}{U}_2 + \overset{12}{I}\,\overset{2}{U}_2 + \overset{13}{I}\,\overset{3}{U}_2 = 0,\\
&\overset{1}{U}_3{}^{\bullet\bullet} + \overset{11}{I}\,\overset{1}{U}_3 + \overset{12}{I}\,\overset{2}{U}_3 + \overset{13}{I}\,\overset{3}{U}_3 = 0,\\
&\overset{2}{U}_1{}^{\bullet\bullet} + \overset{21}{I}\,\overset{1}{U}_1 + \overset{22}{I}\,\overset{2}{U}_1 + \overset{23}{I}\,\overset{3}{U}_1 = 0,\\
&\overset{2}{U}_2{}^{\bullet\bullet} + \overset{21}{I}\,\overset{1}{U}_2 + \overset{22}{I}\,\overset{2}{U}_2 + \overset{23}{I}\,\overset{3}{U}_2 = 0,\\
&\overset{2}{U}_3{}^{\bullet\bullet} + \overset{21}{I}\,\overset{1}{U}_3 + \overset{22}{I}\,\overset{2}{U}_3 + \overset{23}{I}\,\overset{3}{U}_3 = 0,\\
&\overset{3}{U}_1{}^{\bullet\bullet} + \overset{31}{I}\,\overset{1}{U}_1 + \overset{32}{I}\,\overset{2}{U}_1 + \overset{33}{I}\,\overset{3}{U}_1 = 0,\\
&\overset{3}{U}_2{}^{\bullet\bullet} + \overset{31}{I}\,\overset{1}{U}_2 + \overset{32}{I}\,\overset{2}{U}_2 + \overset{33}{I}\,\overset{3}{U}_2 = 0,\\
&\overset{3}{U}_3{}^{\bullet\bullet} + \overset{31}{I}\,\overset{1}{U}_3 + \overset{32}{I}\,\overset{2}{U}_3 + \overset{33}{I}\,\overset{3}{U}_3 = 0.
\end{aligned}
\tag{82}
$$

In the system of equations (82) the subscripts number Cartesian coordinates of the vector $\overset{a}{\mathbf{U}}$; the over-letter indices are Yang-Mills indices.
The explicit notation of the components of YM-inertia tensor ***tymos***, according to (76), has the following form:

$$
\begin{aligned}
&\overset{11}{I} = \overset{2}{\mathbf{U}}{}^2 + \overset{3}{\mathbf{U}}{}^2 = \overset{2}{U}_1{}^2 + \overset{2}{U}_2{}^2 + \overset{2}{U}_3{}^2 + \overset{3}{U}_1{}^2 + \overset{3}{U}_2{}^2 + \overset{3}{U}_3{}^2,\\
&\overset{22}{I} = \overset{3}{\mathbf{U}}{}^2 + \overset{1}{\mathbf{U}}{}^2 = \overset{3}{U}_1{}^2 + \overset{3}{U}_2{}^2 + \overset{3}{U}_3{}^2 + \overset{1}{U}_1{}^2 + \overset{1}{U}_2{}^2 + \overset{1}{U}_3{}^2,\\
&\overset{33}{I} = \overset{1}{\mathbf{U}}{}^2 + \overset{2}{\mathbf{U}}{}^2 = \overset{1}{U}_1{}^2 + \overset{1}{U}_2{}^2 + \overset{1}{U}_3{}^2 + \overset{2}{U}_1{}^2 + \overset{2}{U}_2{}^2 + \overset{2}{U}_3{}^2,\\
&\overset{12}{I} = \overset{21}{I} = -\,\overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}} = -\left(\overset{1}{U}_1\overset{2}{U}_1 + \overset{1}{U}_2\overset{2}{U}_2 + \overset{1}{U}_3\overset{2}{U}_3\right),\\
&\overset{13}{I} = \overset{31}{I} = -\,\overset{1}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}} = -\left(\overset{1}{U}_1\overset{3}{U}_1 + \overset{1}{U}_2\overset{3}{U}_2 + \overset{1}{U}_3\overset{3}{U}_3\right),\\
&\overset{23}{I} = \overset{32}{I} = -\,\overset{2}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}} = -\left(\overset{2}{U}_1\overset{3}{U}_1 + \overset{2}{U}_2\overset{3}{U}_2 + \overset{2}{U}_3\overset{3}{U}_3\right).
\end{aligned}
\tag{83}
$$

The explicit notation of energy integral (81) has the following form:

$$
\begin{aligned}
&\mathfrak{K}+\mathfrak{U}=1,\\
&\mathfrak{K}=\frac{1}{2}\Bigg(\Big(\overset{1}{\mathrm{U}}_1{}^{\bullet}\Big)^2+\Big(\overset{1}{\mathrm{U}}_2{}^{\bullet}\Big)^2+\Big(\overset{1}{\mathrm{U}}_3{}^{\bullet}\Big)^2+\Big(\overset{2}{\mathrm{U}}_1{}^{\bullet}\Big)^2+\\
&\qquad+\Big(\overset{2}{\mathrm{U}}_2{}^{\bullet}\Big)^2+\Big(\overset{2}{\mathrm{U}}_3{}^{\bullet}\Big)^2+\Big(\overset{3}{\mathrm{U}}_1{}^{\bullet}\Big)^2+\Big(\overset{3}{\mathrm{U}}_2{}^{\bullet}\Big)^2+\Big(\overset{3}{\mathrm{U}}_3{}^{\bullet}\Big)^2\Bigg),\\
&\mathfrak{U}=\frac{1}{2}\Bigg(\Big(\overset{1}{\mathrm{U}}_1\overset{2}{\mathrm{U}}_2-\overset{1}{\mathrm{U}}_2\overset{2}{\mathrm{U}}_1\Big)^2+\Big(\overset{2}{\mathrm{U}}_1\overset{3}{\mathrm{U}}_2-\overset{2}{\mathrm{U}}_2\overset{3}{\mathrm{U}}_1\Big)^2+\Big(\overset{1}{\mathrm{U}}_1\overset{3}{\mathrm{U}}_2-\overset{1}{\mathrm{U}}_2\overset{3}{\mathrm{U}}_1\Big)^2+\\
&\qquad+\Big(\overset{1}{\mathrm{U}}_1\overset{2}{\mathrm{U}}_3-\overset{1}{\mathrm{U}}_3\overset{2}{\mathrm{U}}_1\Big)^2+\Big(\overset{2}{\mathrm{U}}_1\overset{3}{\mathrm{U}}_3-\overset{2}{\mathrm{U}}_3\overset{3}{\mathrm{U}}_1\Big)^2+\Big(\overset{1}{\mathrm{U}}_1\overset{3}{\mathrm{U}}_3-\overset{1}{\mathrm{U}}_3\overset{3}{\mathrm{U}}_1\Big)^2+\\
&\qquad+\Big(\overset{1}{\mathrm{U}}_2\overset{2}{\mathrm{U}}_3-\overset{1}{\mathrm{U}}_3\overset{2}{\mathrm{U}}_2\Big)^2+\Big(\overset{2}{\mathrm{U}}_2\overset{3}{\mathrm{U}}_3-\overset{2}{\mathrm{U}}_3\overset{3}{\mathrm{U}}_2\Big)^2+\Big(\overset{1}{\mathrm{U}}_2\overset{3}{\mathrm{U}}_3-\overset{1}{\mathrm{U}}_3\overset{3}{\mathrm{U}}_2\Big)^2\Bigg).
\end{aligned}
\tag{84}
$$

The explicit form of three nonholonomic constraints (73) notation looks as follows:

$$
\begin{aligned}
&\overset{2}{\mathbf{U}}\cdot\overset{3}{\mathbf{U}}{}^{\bullet}-\overset{3}{\mathbf{U}}\cdot\overset{2}{\mathbf{U}}{}^{\bullet}=\Big(\overset{2}{\mathrm{U}}_1\overset{3}{\mathrm{U}}_1{}^{\bullet}+\overset{2}{\mathrm{U}}_2\overset{3}{\mathrm{U}}_2{}^{\bullet}+\overset{2}{\mathrm{U}}_3\overset{3}{\mathrm{U}}_3{}^{\bullet}\Big)-\Big(\overset{3}{\mathrm{U}}_1\overset{2}{\mathrm{U}}_1{}^{\bullet}+\overset{3}{\mathrm{U}}_2\overset{2}{\mathrm{U}}_2{}^{\bullet}+\overset{3}{\mathrm{U}}_3\overset{2}{\mathrm{U}}_3{}^{\bullet}\Big)=0,\\
&\overset{3}{\mathbf{U}}\cdot\overset{1}{\mathbf{U}}{}^{\bullet}-\overset{1}{\mathbf{U}}\cdot\overset{3}{\mathbf{U}}{}^{\bullet}=\Big(\overset{3}{\mathrm{U}}_1\overset{1}{\mathrm{U}}_1{}^{\bullet}+\overset{3}{\mathrm{U}}_2\overset{1}{\mathrm{U}}_2{}^{\bullet}+\overset{3}{\mathrm{U}}_3\overset{1}{\mathrm{U}}_3{}^{\bullet}\Big)-\Big(\overset{1}{\mathrm{U}}_1\overset{3}{\mathrm{U}}_1{}^{\bullet}+\overset{1}{\mathrm{U}}_2\overset{3}{\mathrm{U}}_2{}^{\bullet}+\overset{1}{\mathrm{U}}_3\overset{3}{\mathrm{U}}_3{}^{\bullet}\Big)=0,\\
&\overset{1}{\mathbf{U}}\cdot\overset{2}{\mathbf{U}}{}^{\bullet}-\overset{2}{\mathbf{U}}\cdot\overset{1}{\mathbf{U}}{}^{\bullet}=\Big(\overset{1}{\mathrm{U}}_1\overset{2}{\mathrm{U}}_1{}^{\bullet}+\overset{1}{\mathrm{U}}_2\overset{2}{\mathrm{U}}_2{}^{\bullet}+\overset{1}{\mathrm{U}}_3\overset{2}{\mathrm{U}}_3{}^{\bullet}\Big)-\Big(\overset{2}{\mathrm{U}}_1\overset{1}{\mathrm{U}}_1{}^{\bullet}+\overset{2}{\mathrm{U}}_2\overset{1}{\mathrm{U}}_2{}^{\bullet}+\overset{2}{\mathrm{U}}_3\overset{1}{\mathrm{U}}_3{}^{\bullet}\Big)=0.
\end{aligned}
\tag{85}
$$

The nonholonomic constraints (85) are the restrictions imposed on derivatives $\overset{a}{\mathbf{U}}{}^{\bullet}$ at the initial time. If they are satisfied at the initial time, they are always satisfied under the equations of motion (82).

It is easy to specify two trivial methods of meeting the constraint conditions (85) at the initial time.

1) Zero initial conditions for all potentials are $\overset{a}{\mathbf{U}}$: $\overset{a}{\mathbf{U}}\,(0)$ at the arbitrary initial conditions for derivatives $\overset{a}{\mathbf{U}}{}^{\bullet}(0)$[25]. Let us name this method ZIPO ("zero-initial-potentials").

2) Zero initial conditions for all derivatives of potentials are $\overset{a}{\mathbf{U}}{}^{\bullet}(0)=0$ at the arbitrary initial conditions for the potentials[26]. Let us name this method ZIDE ("zero-initial-derivatives").

In general case it is possible to cope with a problem of conditions (85) as follows:

(1) To set arbitrarily the trial initial values for all nine component of the three vectors of YM-potential triplet $^{*}\,\overset{a}{\mathrm{U}}_i\,(0)$, $\big(a=\overline{1,\,3};\ i=\overline{1,\,3}\big)$.

(2) To set arbitrarily the trial initial values of six "non-diagonal" components of the derivatives from time potentials $^{*}\,\overset{a}{\mathrm{U}}_i{}^{\bullet}$ $(a\neq i)$.

(3) To solve the system of equations (85) as a system of the three linear equations relative to the three "missing" initial values of the derivatives from time potentials

[25] Excepting the requirement of unit normalization by energy *tymos*.

[26] The proviso, restricting this arbitrary rule, is the same as for ZIPO.

$* \overset{a}{\mathrm{U}}_i{}^{\bullet}(0)$ $(a = i)$.

It is easy to demonstrate that the condition for this linear system solvability is the following condition:

$$\Delta = \overset{1}{\mathrm{U}}_2 \overset{2}{\mathrm{U}}_3 \overset{3}{\mathrm{U}}_1 - \overset{2}{\mathrm{U}}_1 \overset{3}{\mathrm{U}}_2 \overset{1}{\mathrm{U}}_2 \neq 0,$$

imposed on the "trial" initial values of potentials $* \overset{a}{\mathrm{U}}_i$.

(4) To calculate the "trial" energy value $^*\mathrm{E}$ through formulas (84) for kinetic energy $\mathfrak{K}$ and potential energy $\mathfrak{U}$. This value will be arbitrary, and will not be equal to unit.

(5) To set the "correct" initial conditions $\overset{a}{\mathrm{U}}_i\,(0)$ and $\overset{a}{\mathrm{U}}_i{}^{\bullet}(0)$ by means of scale transformation of the "trial" initial conditions:

$$\overset{a}{\mathrm{U}}_i\,(0) = {}^*\overset{a}{\mathrm{U}}_i\,(0) \cdot ({}^*\mathrm{E})^{-\frac{1}{4}},$$

$$\overset{a}{\mathrm{U}}_i{}^{\bullet}(0) = {}^*\overset{a}{\mathrm{U}}_i{}^{\bullet}(0) \cdot ({}^*\mathrm{E})^{-\frac{1}{2}}.$$

With these "correct" initial conditions the equations of nonholonomic constraints (85) will be satisfied and tymos energy will be normalized to a unit.

The positive definiteness of diagonal elements of matrix $\overset{ab}{\mathrm{I}}$ (83) and the positive definiteness of potential energy $\mathfrak{U}$ (84) allow to claim that the solution to a nonlinear system of the ordinary differential equations of the 18th order (82), describing *tymos* dynamics, have oscillating character. Numerical solutions of the system (82) which will be presented in one of the subsequent articles of this series show that these ostsillations have a difficult, almost unpredictable character. Dependences of potential components $\overset{a}{\mathrm{U}}_i$ on time are subjected to a "butterfly effect". All reliable data on *tymos* dynamics is concentrated in statistical characteristics of these dependences. The conservative system (82) does not have attractors. The solution trajectory gradually fills some area in the nine-dimensional configurational space $\overset{a}{\mathrm{U}}_i$. The density of this filling is an important characteristic of *tymos* dynamics. It is convenient to use energy integral (84) and nonholonomic constraints (85) as a prover of accuracy and stability of calculations in numerical solution of the differential equation system (82).

9.2.7 Chirality problem

Yang-Mills-Euler equations (82) are chiral-symmetric, i.e. they do not distinguish the right-hand and the left-hand. Their solutions are also chiral-symmetric. But the original variables of the free triplet field Lagrangian (50) have no chiral symmetry. This problem has been previously mentioned, in p.4.1.

The very fact of the presence of Yang-Mills' vector product in the triplet theory means that we have no right to renumber the vectors in YM-triple arbitrarily: this triple must have a fixed orientation. In particular, for the problem on the plane wave, considered here, it means that orientation of the triple of three-dimensional vectors $\overset{a}{\mathbf{U}}$ must coincide with the chosen orientation of the three-dimensional spatial coordinate system ("the principle of chiral consistency"). It is convenient to watch the triple orientation $\overset{a}{\mathbf{U}}$ by the sign of the mixed scalar-vector product of three vectors $\overset{a}{\mathbf{U}}$. Let us call this product a "chiral determinant" CD:

$$\mathrm{CD}\left(\overset{1}{\mathbf{U}}, \overset{2}{\mathbf{U}}, \overset{3}{\mathbf{U}}\right) = \left(\overset{1}{\mathbf{U}} \times \overset{2}{\mathbf{U}}\right) \cdot \overset{3}{\mathbf{U}} = \overset{1}{\mathbf{U}} \cdot \left(\overset{2}{\mathbf{U}} \times \overset{3}{\mathbf{U}}\right). \tag{86}$$

The chirality determinacy condition, or to be short, "chirality condition" looks as follows:

$$\mathrm{CD}\left(\overset{1}{\mathbf{U}}, \overset{2}{\mathbf{U}}, \overset{3}{\mathbf{U}}\right) \geq 0. \tag{87}$$

The value of CD (86) is not Lorentz-invariant, since the procedure of separation of the components of 4-potential vector $\overset{a}{\mathrm{A}}{}^{\mu}$ into time and space parts is not Lorentz-invariant. But CD sign (and it is only this sign that is important in chiral condition (87)) is Lorentz-invariant.

We can control the sign of CD, calculating CD in any Lorentz system, for example, in the intrinsic frame of the triplet wave.

The initial conditions for the system (82) must satisfy the condition (87). In addition, the condition (87) should be satisfied at any time. However, the continuous smooth solutions of Yang-Mills-Euler equations (82) do not obey to the chiral condition (87): the value of CD oscillates in time, changing the sign. It is possible to satisfy the condition (87) only by sacrificing the continuity or smoothness of the solutions.

The method, based on the loss of continuity of the solution, consists of permutation of vectors $\overset{1}{\mathbf{U}}$ and $\overset{2}{\mathbf{U}}$ when $\mathrm{CD} = 0$. Component $\overset{3}{\mathbf{U}}$ should not be affected by this permutation on the following reasons. The third YM-component of YM-current is coupled with the singlet current. In other words, the third component of the YM-triplet vector is initially distinguished and "unequal" with the components number 1 and number 2. Components 1 and 2 are equal in the condition of inner normalization of the triplet of currents (32) and differ only in the condition of chiral determinacy (87). When this condition is violated, these vectors should be just rearranged to satisfy the condition again. Let us name this rearrangement "the hard chiralization of solution".

Dwelling on the reasonableness of the "hard chiralization" in the framework of classical (non-quantum) physics would be as useless as dwelling on the reasonableness of Pauli Exclusion Principle in non-relativistic quantum mechanics: for non-relativistic theory, Pauli principle is a phenomenological principle which is a part of the theory, but has no justification within the theory. The procedure of "hard chiralization" within the framework of non-quantum theory should also be taken as a phenomenological rule. "Hard chiralization" generates discontinuous solutions, which are as chaotic as continuous and smooth chiralo-symmetric solutions of the equations (82). Actually, these are the same chiral -symmetric solutions, but they have been re-interpreted by the sporadic permutations $1 \leftrightarrow 2$ in YM-indices.

However, this "hard chiralization", a shocking but a valid method to provide chiral determinacy of the solutions, acceptable in the triplet sector of physics, is inadmissible for the octuplet sector. In order to ensure the chiral definiteness in the group of eight Yang-Mills vectors of the octuplet wave (such wave is a classical model of a quantum object, gluon), it is necessary to provide a vector permutation in several triples of vectors simultaneously (the structure of these triples is no more determined by Levi-Civita symbol, but by the structure constants of the group SU(3),specifying the analogue of vector product in the eight-dimensional YM-space), which is found to be impossible.

Therefore, let us turn to studying another method for providing chiral determinacy (87), which is based on the use of continuous, but not smooth, solutions.

This method, which we will call "the soft chiralization", is generated by the possibility of mechanical interpretation of the condition (87) as a one-sided, unilateral constraint, imposed on the equations solution (82).

Such constraint, set by inequality, appears, for example, in the problem of the plane motion of mathematical pendulum on nonstretchable line. If the line is stretched, the point pendulum swings, moving along the circular arc of the radius equal to the line's length. But if the line is not stretched, the point pendulum, as a free material point, moves along the arc of parabola in the homogeneous gravitational field. At the moments when the line stretches, the radial velocity component of the pendulum reverses the sign abruptly, the azimuthal velocity component remains the same. If there is no dissipation in the line, such swings of the pendulum along the parabolic trajectories continue for unlimitedly long time.

Based on this mechanical analogy, we will formulate the following method for "chiralizat-sion" of the solution: at the moments when CD $\left(\overset{1}{\mathbf{U}}, \overset{2}{\mathbf{U}}, \overset{3}{\mathbf{U}}\right) = 0$, the signs of all velocity vectors $\overset{a}{\mathbf{U}}{}^{\bullet}$ are reversed.

This method of "soft chiralization" generates continuous, but not smooth, chiral-determined solutions: the condition of the chiral determinacy is always satisfied; and at the moment of "nulling" of the chiral CD determinant (86), the velocities $\overset{a}{\mathbf{U}}{}^{\bullet}$ discontinue.

This method of "soft chiralization" is literally transferred into the octuplet sector of physics: at "nulling" of at least one of the set of chiral determinants of the octuplet sector, the signs of rates of change for all eight Yang-Mills octuplet vectors are reversed. As it was previously noted in p. 4.1., in the problem of mathematical pendulum motion, it is possible to get rid of the solutions with velocity discontinuity by introducing a more complex model of constraint, for example, taking into account the resilient extensibility of the fiber on which the pendulum is hung. There is no such possibility in the problems of Yang-Mills physics: we have no more accurate physics beyond Yang-Mills physics. We have to put up with the roughness of the chiral-determined YM-solutions. These discontinuities in derivatives mean a sudden, instantaneous occurrence and vanishing of YM-currents in a zero-current problem. The dependence of these currents on time is described by δ-delta function.

In general, CD disappears when the three YM-vectors $\overset{a}{\mathbf{U}}$ are in the same plane. This is a "reflection plane" – hitting against it, all three vectors $\overset{a}{\mathbf{U}}$ rebound resiliently, providing chiral definiteness of the solutions for the equations (82). Two of such fixed "planes of reflection" appear at the arbitrary initial conditions, and the points representing the termini of the three YM-vectors $\overset{a}{\mathbf{U}}$, make periodic motions between the two planes. Numerical solutions of the equations (82), accounting the conditions of the chiral determinacy (87), will be presented later in one of the articles of this series.

9.2.8 Yang-Mills Orthogonal Zero-current Triplet Oscillator

The problem of Yang-Mills plane wave takes especially simple and attractive form if we assume that

$$\overset{a}{\mathrm{U}}_i = 0, \quad \text{with} \quad i \neq a. \tag{88}$$

The condition (88) means that all three Yang-Mills vectors $\overset{a}{\mathbf{U}}$ are orthogonal to one another and each of them has a stationary direction, which, according to (88), we will identify with the direction of the corresponding coordinate axis of the three-dimensional Euclidean space in the intrinsic wave system.

When the condition of orthogonality (88) is fulfilled, the equations of non-holonomic constraints (85) are identically satisfied and YM-inertia tensor (83) becomes diagonal. Taking into account the condition (88), we no longer need to use two kinds of indices – the over-letter YM-indices, labeling the vectors of YM-triplet, and the subscripts labeling the spatial Cartesian components of each YM-triplet vector.

In the problems of *tymos* dynamics, considering the orthogonality condition (88), there remain only three non-trivial dynamic variables which have the same Yang-Mills and spatial indices. In the process it is convenient to use one- index notation system, assuming that

$$u_i \equiv \overset{a}{\mathrm{U}}_i \quad \left(i = a = \overline{1,\, 3}\right). \tag{89}$$

In this recording Yang-Mills-Euler equations (82) take the following "canonical" form:

$$\begin{aligned} \ddot{u}_1 + u_1\left(u_2^2 + u_3^2\right) &= 0, \\ \ddot{u}_2 + u_2\left(u_3^2 + u_1^2\right) &= 0, \\ \ddot{u}_3 + u_3\left(u_1^2 + u_2^2\right) &= 0. \end{aligned} \tag{90}$$

In such one-index notation, due to the absence of explicit YM-indices, it is proper to put a dot which labels the intrinsic wave time derivative, over the corresponding letter.

Energy integral (84) for the system (90) also takes an elegant "canonical" form:

$$\frac{1}{2}\left((\dot{u}_1)^2 + (\dot{u}_2)^2 + (\dot{u}_3)^2\right) + \frac{1}{2}\left(u_1^2\, u_2^2 + u_2^2\, u_3^2 + u_3^2\, u_1^2\right) = 1. \tag{91}$$

Let us name the three-dimensional system of motion (90) with energy integral (91) a "Yang-Mills orthogonal triplet oscillator", denoting it with "$\perp -$*tymos*". With certain conditionality we can treat the three one-index values (89) as three Cartesian components of one vector depicting *tymos* in a three-dimensional configuration space, which allows us to treat $\perp -$*tymos* as a "one-vector *tymos*" (1-vector-tymos).

Assuming that in (90) one of the components, for example u_3, is identically zero, we get even simpler problem of two-dimensional *tymos*:

$$\begin{aligned} \ddot{u}_1 + u_1 u_2^2 &= 0, \\ \ddot{u}_2 + u_2 u_1^2 &= 0, \end{aligned} \tag{92}$$

with the energy integral:

$$\frac{1}{2}\left((\dot{u}_1)^2 + (\dot{u}_2)^2\right) + \frac{1}{2}\,u_1^2\,u_2^2 = 1. \tag{93}$$

Let us call the system of motion (92) with energy integral (93) a "doubletic Yang-Mills' oscillator", ***dymos***.
The essential nonlinearity of the equations (90) or (92), as well as of the whole Yang-Mills wave theory should be noted. Such problems in principle do not allow any interpretation within the framework of the perturbation theory. The linearization of these problems by neglecting nonlinear terms in (90) or (92) leads to such significant degeneracy of the problem that it can not be corrected by constructing a chain of successive approximations.

9.2.9 Yang-Mills Plane Zero-current Wave with Space-like Wave Vector

There are no a priori reasons that allow to regard Yang-Mills waves only with time-like wave vector.
Let us make the transformations, made previously in pp. 9.2.2. – 9.2.6. and p. 9.2.8. again, now supposing that the wave vector of the plane zero-current wave k^μ is space-like:

$$k^\mu k_\mu = -\varepsilon^2 < 0.$$

After performing the scale transformation (p. 9.2.2) which normalizes the wave vector per unit, and the subsequent scale transformation (p. 9.2.3) which normalizes the wave energy per unit[27], assuming that there is not any stationary field external to the wave, we can reduce the problem of Yang-Mills plane zero-current triplet wave with a space-like wave vector k^μ to the following form:

$$\begin{aligned}
&\overset{a}{\mathrm{A}}{}^{\nu\prime\prime} - \overset{ab}{\mathrm{I}}\,\overset{b}{\mathrm{A}}{}^{\nu} = 0,\\
&\overset{abc}{\varepsilon}\,\overset{b}{\mathrm{A}}{}^{\mu}\,\overset{c}{\mathrm{A}}{}'_{\mu} = 0,\\
&k^\mu\,\overset{a}{\mathrm{A}}_{\mu} = 0,\\
&k^\mu\,k_\mu = -1.
\end{aligned} \tag{94}$$

Yang-Mills inertia tensor $\overset{ab}{\mathrm{I}}$ is determined by formulas (75) and (77).
The energy integral for system of motion (94) differs from formula (61) which is fair for the wave with time-like wave vector k^μ, in "the wrong" sign in "kinetic" energy $\mathfrak{K}$ and in the absence of energy E sign-definiteness:

$$\frac{1}{2}\left(\overset{a}{\mathrm{A}}{}'_{\nu}\cdot\overset{a}{\mathrm{A}}{}^{\nu\prime}\right) + \frac{1}{4}\left(\left(\overset{a}{\mathrm{A}}_{\nu}\cdot\overset{a}{\mathrm{A}}{}^{\nu}\right)^2 - \left(\overset{a}{\mathrm{A}}_{\mu}\cdot\overset{a}{\mathrm{A}}_{\nu}\right)\left(\overset{b}{\mathrm{A}}{}^{\mu}\cdot\overset{b}{\mathrm{A}}{}^{\nu}\right)\right) = \mathrm{sign}\,\mathrm{E} = \pm\,1.^{28} \tag{95}$$

[27] In this transformation, in contrast to formulas (62) which are fair for the wave with a time-like wave vector k^μ and sign-definite energy E, it is not energy E that appears, but its module $|\,\mathrm{E}\,|$, since energy E loses its sign-definiteness for the wave with space-like wave vector k^μ. A particular case $\mathrm{E} = 0$ must be considered separately.
[28] The wave under consideration also can have zero energy.

In relations (94) and (28) the sign $'$ on the right side from potential $\overset{a}{\mathrm{A}}{}^{\nu}$ means the wave phase ϕ derivative.
Dynamical system (94) with "the wrong" sign does not have any oscillate solutions, in contrast to the system of motion (79).

9.2.10 Yang-Mills Plane Zero-current Wave with the Space-like Wave Vector in the Intrinsic Frame of Reference

Let us call the frame of reference, in which a space-like wave vector, normalized per unit, has the following form, the intrinsic frame of reference:

$$k^{\mu} = \{0;\ 1;\ 0;\ 0\}.$$

In this frame of reference, axis x is directed along the wave vector, and the wave phase derivative coincides with the arbitrary coordinate x derivative. The wave in this frame of reference is static, its characteristics do not depend on time.
Longitudinal components of 4-potential triplet in this frame of reference are equal to zero. The notation of the potentials in the form "1+3" (70) looks as follows:

$$\overset{a}{\mathrm{A}}{}^{\nu} = \{\overset{a}{\mathrm{T}};\ 0;\ \overset{a}{\mathrm{u}}_2;\ \overset{a}{\mathrm{u}}_3\}.$$

Substitution of this formula into the equations of motion (94) results in the following nine-component dynamical problem relative to the nine nontrivial components of Yang-Mills triplet:

$$\begin{aligned}
&-\overset{1}{\mathrm{T}}{}''+\overset{11}{\mathrm{I}}\,\overset{1}{\mathrm{T}}+\overset{12}{\mathrm{I}}\,\overset{2}{\mathrm{T}}+\overset{13}{\mathrm{I}}\,\overset{3}{\mathrm{T}}=0,\\
&-\overset{1}{\mathrm{u}}_2{}''+\overset{11}{\mathrm{I}}\,\overset{1}{\mathrm{u}}_2+\overset{12}{\mathrm{I}}\,\overset{2}{\mathrm{u}}_2+\overset{13}{\mathrm{I}}\,\overset{3}{\mathrm{u}}_2=0,\\
&-\overset{1}{\mathrm{u}}_3{}''+\overset{11}{\mathrm{I}}\,\overset{1}{\mathrm{u}}_3+\overset{12}{\mathrm{I}}\,\overset{2}{\mathrm{u}}_3+\overset{13}{\mathrm{I}}\,\overset{3}{\mathrm{u}}_3=0,\\
&-\overset{2}{\mathrm{T}}{}''+\overset{21}{\mathrm{I}}\,\overset{1}{\mathrm{T}}+\overset{22}{\mathrm{I}}\,\overset{2}{\mathrm{T}}+\overset{23}{\mathrm{I}}\,\overset{3}{\mathrm{T}}=0,\\
&-\overset{2}{\mathrm{u}}_2{}''+\overset{21}{\mathrm{I}}\,\overset{1}{\mathrm{u}}_2+\overset{22}{\mathrm{I}}\,\overset{2}{\mathrm{u}}_2+\overset{23}{\mathrm{I}}\,\overset{3}{\mathrm{u}}_2=0,\\
&-\overset{2}{\mathrm{u}}_3{}''+\overset{21}{\mathrm{I}}\,\overset{1}{\mathrm{u}}_3+\overset{22}{\mathrm{I}}\,\overset{2}{\mathrm{u}}_3+\overset{23}{\mathrm{I}}\,\overset{3}{\mathrm{u}}_3=0,\\
&-\overset{3}{\mathrm{T}}{}''+\overset{31}{\mathrm{I}}\,\overset{1}{\mathrm{T}}+\overset{32}{\mathrm{I}}\,\overset{2}{\mathrm{T}}+\overset{33}{\mathrm{I}}\,\overset{3}{\mathrm{T}}=0,\\
&-\overset{3}{\mathrm{u}}_2{}''+\overset{31}{\mathrm{I}}\,\overset{1}{\mathrm{u}}_2+\overset{32}{\mathrm{I}}\,\overset{2}{\mathrm{u}}_2+\overset{33}{\mathrm{I}}\,\overset{3}{\mathrm{u}}_2=0,\\
&-\overset{3}{\mathrm{u}}_3{}''+\overset{31}{\mathrm{I}}\,\overset{1}{\mathrm{u}}_3+\overset{32}{\mathrm{I}}\,\overset{2}{\mathrm{u}}_3+\overset{33}{\mathrm{I}}\,\overset{3}{\mathrm{u}}_3=0.
\end{aligned} \tag{96}$$

The components of Yang-Mills inertia tensor $\overset{ab}{\mathrm{I}}$ in (96) have the following form:

$$\begin{aligned}
&\overset{11}{\mathrm{I}}= \left(\overset{2}{\mathrm{u}}_2\right)^2 + \left(\overset{2}{\mathrm{u}}_3\right)^2 + \left(\overset{3}{\mathrm{u}}_2\right)^2 + \left(\overset{3}{\mathrm{u}}_3\right)^2 - \left(\overset{2}{\mathrm{T}}\right)^2 - \left(\overset{3}{\mathrm{T}}\right)^2;\\
&\overset{22}{\mathrm{I}}= \left(\overset{3}{\mathrm{u}}_2\right)^2 + \left(\overset{3}{\mathrm{u}}_3\right)^2 + \left(\overset{1}{\mathrm{u}}_2\right)^2 + \left(\overset{1}{\mathrm{u}}_3\right)^2 - \left(\overset{3}{\mathrm{T}}\right)^2 - \left(\overset{1}{\mathrm{T}}\right)^2;\\
&\overset{33}{\mathrm{I}}= \left(\overset{1}{\mathrm{u}}_2\right)^2 + \left(\overset{1}{\mathrm{u}}_3\right)^2 + \left(\overset{2}{\mathrm{u}}_2\right)^2 + \left(\overset{2}{\mathrm{u}}_3\right)^2 - \left(\overset{1}{\mathrm{T}}\right)^2 - \left(\overset{2}{\mathrm{T}}\right)^2;\\
&\overset{12}{\mathrm{I}}=\overset{21}{\mathrm{I}}=\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{T}} - \left(\overset{1}{\mathrm{u}}_2\,\overset{2}{\mathrm{u}}_2 + \overset{1}{\mathrm{u}}_3\,\overset{2}{\mathrm{u}}_3\right);\\
&\overset{23}{\mathrm{I}}=\overset{32}{\mathrm{I}}=\overset{2}{\mathrm{T}}\,\overset{3}{\mathrm{T}} - \left(\overset{2}{\mathrm{u}}_2\,\overset{3}{\mathrm{u}}_2 + \overset{2}{\mathrm{u}}_3\,\overset{3}{\mathrm{u}}_3\right);\\
&\overset{31}{\mathrm{I}}=\overset{13}{\mathrm{I}}=\overset{3}{\mathrm{T}}\,\overset{1}{\mathrm{T}} - \left(\overset{3}{\mathrm{u}}_2\,\overset{1}{\mathrm{u}}_2 + \overset{3}{\mathrm{u}}_3\,\overset{1}{\mathrm{u}}_3\right).
\end{aligned} \tag{97}$$

The explicit notation of non-holonomic constraints (67), (68), imposed on the solution of the system (96), has the following form:

$$\begin{aligned}
&\left(\overset{2}{\mathrm{T}}\,\overset{3}{\mathrm{T}}{}' - \overset{3}{\mathrm{T}}\,\overset{2}{\mathrm{T}}{}'\right) - \left(\overset{2}{\mathrm{u}}_2\,\overset{3}{\mathrm{u}}{}'_2 + \overset{2}{\mathrm{u}}_3\,\overset{3}{\mathrm{u}}{}'_3\right) - \left(\overset{3}{\mathrm{u}}_2\,\overset{2}{\mathrm{u}}{}'_2 + \overset{3}{\mathrm{u}}_3\,\overset{2}{\mathrm{u}}{}'_3\right) = 0,\\
&\left(\overset{3}{\mathrm{T}}\,\overset{1}{\mathrm{T}}{}' - \overset{1}{\mathrm{T}}\,\overset{3}{\mathrm{T}}{}'\right) - \left(\overset{3}{\mathrm{u}}_2\,\overset{1}{\mathrm{u}}{}'_2 + \overset{3}{\mathrm{u}}_3\,\overset{1}{\mathrm{u}}{}'_3\right) - \left(\overset{1}{\mathrm{u}}_2\,\overset{3}{\mathrm{u}}{}'_2 + \overset{1}{\mathrm{u}}_3\,\overset{3}{\mathrm{u}}{}'_3\right) = 0,\\
&\left(\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{T}}{}' - \overset{2}{\mathrm{T}}\,\overset{1}{\mathrm{T}}{}'\right) - \left(\overset{1}{\mathrm{u}}_2\,\overset{2}{\mathrm{u}}{}'_2 + \overset{1}{\mathrm{u}}_3\,\overset{2}{\mathrm{u}}{}'_3\right) - \left(\overset{2}{\mathrm{u}}_2\,\overset{1}{\mathrm{u}}{}'_2 + \overset{2}{\mathrm{u}}_3\,\overset{1}{\mathrm{u}}{}'_3\right) = 0.
\end{aligned} \tag{98}$$

Non-holonomic constraints (98) are actually imposed only on the initial values of the variables with some arbitrary initial value of the longitudinal coordinate x. If these conditions are fulfilled at one value of x, they will be fulfilled at all values of x on the ground of the equations (96).

A trivial method of accounting the conditions (98) consists of setting zero initial conditions for all the nine non-trivial components of triplet of potential or for all of the nine potential derivatives along the longitudinal coordinate x. A less trivial method is similar to the one described above in p 9.2.6 for a wave with a time-like wave vector. It has to solve the constraints, linear by derivatives, relative to any three initial derivatives, considering the other six trial initial values of the derivatives and all the nine trial initial components of the potentials. For example, so that the three equations (98) were solvable relative to the initial values $\overset{1}{\mathrm{T}}{}'$, $\overset{2}{\mathrm{u}}{}'_2$, $\overset{3}{\mathrm{u}}{}'_3$, the corresponding determinant Δ must be nonzero:

$$\Delta = \overset{1}{\mathrm{u}}_2\,\overset{2}{\mathrm{u}}_3\,\overset{3}{\mathrm{T}} - \overset{1}{\mathrm{u}}_3\,\overset{3}{\mathrm{u}}_2\,\overset{2}{\mathrm{T}} \neq 0.$$

This condition must be fulfilled for the trial initial values of the potential components. Then, it is necessary to calculate the trial wave energy E by formula (28) and to rescale the initial values of the potentials and their derivatives in such a way that the wave energy was equal to +1 (if $\mathrm{E} > 0$) or 1 (if $\mathrm{E} < 0$). In the process, it is convenient to use an explicit, but lengthy, expression for energy in the notation "1+3" (70):

$$
\begin{aligned}
&\mathrm{E} = \mathfrak{K} + \mathfrak{U} = \pm 1,\\
&\mathfrak{K} = \frac{1}{2}\Bigg(\left(\overset{1}{\mathrm{T}}{}'\right)^2 + \left(\overset{2}{\mathrm{T}}{}'\right)^2 + \left(\overset{3}{\mathrm{T}}{}'\right)^2 -\\
&\quad - \left(\left(\overset{1}{\mathrm{u}}{}'_2\right)^2 + \left(\overset{1}{\mathrm{u}}{}'_3\right)^2 + \left(\overset{2}{\mathrm{u}}{}'_2\right)^2 + \left(\overset{2}{\mathrm{u}}{}'_3\right)^2 + \left(\overset{3}{\mathrm{u}}{}'_2\right)^2 + \left(\overset{3}{\mathrm{u}}{}'_3\right)^2\right)\Bigg),\\
&\mathfrak{U} = \frac{1}{2}\Bigg(\left(\left(\overset{1}{\mathrm{u}}_2\overset{2}{\mathrm{u}}_3 - \overset{1}{\mathrm{u}}_3\overset{2}{\mathrm{u}}_2\right)^2 + \left(\overset{1}{\mathrm{u}}_2\overset{3}{\mathrm{u}}_3 - \overset{1}{\mathrm{u}}_3\overset{3}{\mathrm{u}}_2\right)^2 + \left(\overset{2}{\mathrm{u}}_2\overset{3}{\mathrm{u}}_3 - \overset{2}{\mathrm{u}}_3\overset{3}{\mathrm{u}}_2\right)^2\right) -\\
&\quad - \Bigg(\left(\overset{1}{\mathrm{T}}\overset{2}{\mathrm{u}}_2 - \overset{2}{\mathrm{T}}\overset{1}{\mathrm{u}}_2\right)^2 + \left(\overset{1}{\mathrm{T}}\overset{2}{\mathrm{u}}_3 - \overset{2}{\mathrm{T}}\overset{1}{\mathrm{u}}_3\right)^2 + \left(\overset{1}{\mathrm{T}}\overset{3}{\mathrm{u}}_2 - \overset{3}{\mathrm{T}}\overset{1}{\mathrm{u}}_2\right)^2 +\\
&\quad + \left(\overset{1}{\mathrm{T}}\overset{3}{\mathrm{u}}_3 - \overset{3}{\mathrm{T}}\overset{1}{\mathrm{u}}_3\right)^2 + \left(\overset{2}{\mathrm{T}}\overset{3}{\mathrm{u}}_2 - \overset{3}{\mathrm{T}}\overset{2}{\mathrm{u}}_2\right)^2 + \left(\overset{2}{\mathrm{T}}\overset{3}{\mathrm{u}}_3 - \overset{3}{\mathrm{T}}\overset{2}{\mathrm{u}}_3\right)^2\Bigg)\Bigg).
\end{aligned}
\tag{99}
$$

The introduction of Yang-Mills triple of complex three-dimensional vectors $\overset{a}{\boldsymbol{\tau}}$:

$$
\overset{a}{\boldsymbol{\tau}} = \{\overset{a}{\mathrm{T}};\ i\ \overset{a}{\mathrm{u}}_2;\ i\ \overset{a}{\mathrm{u}}_3\} - \tag{100}
$$

allows to enter the energy expression in a more compact and visible form:

$$
\begin{aligned}
\mathfrak{K} &= \frac{1}{2}\left(\left(\overset{1}{\boldsymbol{\tau}}{}'\right)^2 + \left(\overset{2}{\boldsymbol{\tau}}{}'\right)^2 + \left(\overset{3}{\boldsymbol{\tau}}{}'\right)^2\right);\\
\mathfrak{U} &= \frac{1}{2}\left(\left(\overset{1}{\boldsymbol{\tau}} \times \overset{2}{\boldsymbol{\tau}}\right)^2 + \left(\overset{2}{\boldsymbol{\tau}} \times \overset{3}{\boldsymbol{\tau}}\right)^2 + \left(\overset{3}{\boldsymbol{\tau}} \times \overset{1}{\boldsymbol{\tau}}\right)^2\right).
\end{aligned}
$$

The simplicity of these formulas, in the language of triple $\overset{a}{\boldsymbol{\tau}}$, allows to assume that chiral determinant CD for this wave can also be entered in the form of (86) with the replacement $\overset{a}{\mathbf{U}} \to \overset{a}{\boldsymbol{\tau}}$:

$$
\mathrm{CD}\left(\overset{1}{\boldsymbol{\tau}}, \overset{2}{\boldsymbol{\tau}}, \overset{3}{\boldsymbol{\tau}}\right) = \left(\overset{1}{\boldsymbol{\tau}} \times \overset{2}{\boldsymbol{\tau}}\right) \cdot \overset{3}{\boldsymbol{\tau}} = \overset{1}{\boldsymbol{\tau}} \cdot \left(\overset{2}{\boldsymbol{\tau}} \times \overset{3}{\boldsymbol{\tau}}\right) \geq 0. \tag{101}
$$

However, we cannot advance any argument in favor of this formula[29].

As a matter of fact, the nine-component object under consideration, the Yang-Mills plane zero-current triplet wave with space-like wave vector, should not be called a "wave" at all, since the equations (96) have no continuous, smooth, oscillatory solutions; their solutions grow progressively along the longitudinal coordinate x. Oscillations (i.e. wave ridges and decreases) may appear with the use of the above-mentioned (p. 9.2.7.) procedure of "soft chiralization" with the use of chiral determinant (101) which is denoted in the terms of

[29] The only argument that can be advanced consists of the *naturalness* of *the* appearance of vectors $\overset{a}{\boldsymbol{\tau}}$ and expression (100) for CD within Yang-Mills mathematical apparatus. The arguments of mathematical naturalness and simplicity are quite traditional for mathematical physics, for example, while constructing the Lagrangians. However, it is unknown whether such arguments always lead to a true physical theory: perhaps, God does not use any mathematics at all.

complex Yang-Mills vectors $\overset{a}{\boldsymbol{\tau}}$.
The most shocking fact in description of this nine-component "wave" is the possibility of the existence of an object with negative energy. The world-view of any physicist-theorist of post-Einstein epoch is hardly able to cope with the existence of such object. We tend to cautiously assume that such object cannot exist as an independent wave, but it can appear in a couple with some other object that has an excess of positive energy in the process of some third object's decay. Probably, the appearance of objects with negative energy in the apparatus of classical field theory is not a more shocking fact than the appearance in modern quantum theory of virtual particles which temporarily violate the energy conservation law: both the first and the second "strangeness" may finally describe the same circle of phenomena.
We shall call this "strange" nine-component wave with a Latin term "terriculum" (a scarecrow, a fright)[30].

9.2.11 Yang-Mills Orthogonal Terriculum

Instead of a lengthy nine-component Yang-Mills terriculum, described by the system of equations (96), we can consider a three-component object, which is simpler for analysis: an orthogonal terriculum. Let us suppose that vectors $\overset{a}{\boldsymbol{\tau}}$ in the triple (100) are orthogonal to each other and directed along the corresponding Cartesian axes. In other words, only three of nine components of terriculum potential, $\overset{1}{\mathrm{T}}$, $\overset{2}{\mathrm{U}}_2$, $\overset{3}{\mathrm{U}}_3$, are nonzero components (further in this p. 9.2.11, to make the notation simple, the over-letter YM-indices for designating three of these values, can be omitted). With such choice of triple $\overset{a}{\boldsymbol{\tau}}$, the non-holonomic constraints (98) are identically satisfied, and the system of motion equations (96) takes quite a simple form:

$$\begin{aligned} &\mathrm{T}'' - \mathrm{T}\left(\mathrm{U}_2^2 + \mathrm{U}_3^2\right) = 0, \\ &\mathrm{U}_2'' + \mathrm{U}_2\left(\mathrm{T}^2 - \mathrm{U}_3^2\right) = 0, \\ &\mathrm{U}_3'' + \mathrm{U}_3\left(\mathrm{T}^2 - \mathrm{U}_2^2\right) = 0. \end{aligned} \tag{102}$$

Energy integral (99) for the system (102) takes a more visible form:

$$\begin{aligned} &\mathfrak{K} + \mathfrak{U} = \pm 1, \\ &\mathfrak{K} = \frac{1}{2}\left((\mathrm{T}')^2 - (\mathrm{U}_2)^2 - (\mathrm{U}_3)^2\right), \\ &\mathfrak{U} = \frac{1}{2}\left(\mathrm{U}_2^2\mathrm{U}_3^2 - \mathrm{T}^2\left((\mathrm{U}_2)^2 + (\mathrm{U}_3)^2\right)\right). \end{aligned} \tag{103}$$

The condition of chiral definiteness (101) for system (102) takes the following form:

$$\mathrm{T}\mathrm{U}_2\mathrm{U}_3 \leq 0. \tag{104}$$

[30] If terriculum had been discovered by the theorists of 1950s, it would have, undoubtedly, undermined the physicists' interest for Yang-Mills fields. Six decades later, when no one doubts the reality of Yang-Mills fields, the detailed numerical research of terriculum, with the condition of chiralization (101) would be useful and would hardly be able to undermine the trust to Yang-Mills theory.

Without considering the condition (104), chiral-symmetrical solutions (102) (continuous and smooth) have, as it is easy to see in the form (102), a simple, but unacceptable behavior with $x \to \infty$: an unbounded growth of T at more frequent oscillations of U_2 and U_3. The procedure of "soft chiralization" similar to the one described in p. 9.2.7, interrupts the unbounded growth of the variable T and generates oscillatory solutions for the system (102) with derivative discontinuity.
The procedure of "soft chiralization" for solution of the system (102) is the following: for those quantities of the longitudinal coordinate x, under which at least one of the three decision functions T, U_2, U_3 vanishes, (and, correspondingly, by (104), the chiral determinant vanishes) the signs of the derivative from all the three functions are reversed: $T' \to -T'$, $U_2' \to -U_2'$, $U_3' \to -U_3'$.

9.2.12 Yang-Mills Dilemma

The procedure of "soft chiralization" is able to interrupt the unbounded growth of potential components in the terriculum wave, and, thereby, to make this object more acceptable for analysis in theoretical physics. However, the chiralization procedure itself is introduced into theoretical physics for the first time, and, probably, it does not seem reasonably sufficient to the reader (within the framework of *classical* theory, its sufficiency is probably impossible). But if the chiralization procedure is not used, relative to the terriculum waves, we discover two possible modes of behavior:

(1) To reject the possibility of terriculum wave existence itself as a physically unaccepted object; to accept that Fourier decomposition of Yang-Mills potentials must be performed not over the whole four-dimensional space of the wave vectors, but only within the boundaries of 4-cone $k^\mu k_\mu > 0$.

(2) To put up with the fact of terriculum wave existence, supposing that the problem of terriculum "wrong behavior" on the space infinity can be removed by instability of Yang-Mills free zero-current waves in the cone $k^\mu k_\mu < 0$, relative to the build-up of the current states (one-current, two-current, three-current, four-current) and the subsequent production from them of discrete particles with pomerium.

Let us call the choice of one of these lines a "Yang-Mills dilemma".
We can cite a hypothetical "Dirac's dilemma" as a historically remote analogue: after the discovery of solutions with "the wrong" negative energy in Dirac's equation, these solutions could be neglected as "non-physical" (within the framework of the experimentalists' understanding of physics in 1928), and, by doing so, the positron, still not discovered by those experimentalists, could be "missed"; or it could be possible to put up with such solutions and to look for an opportunity to "build" them into the common picture of physical reality.
It is not ruled out that it is the peculiarities of the "monstrous" behavior of the terriculum that have something to do with the processes in which a "weak" triplet interaction "demolishes" heavy leptons and quarks, which per se would be stable in "maxwellian-and-chromodynamical world", the world without triplet interactions.

9.2.13 Yang-Mills Photon: Plane Zero-current Triplet Wave with Isotropic Wave Vector

Let us have a look at the plane triplet wave with isotropic wave vector k^ν:

$$k^\nu k_\nu = 0.$$

Wave vector isotropy means that the phase velocity of a wave is equal to the velocity of light. We shall call this wave (to be precise, the quantum object, hypothetically corresponding to it) "The Yang-Mills' photon" (ymiton).

Ymiton is a degenerate object of Yang-Mills mathematical wave theory. The condition of a wave vector isotropy removes higher derivatives from the wave equation (55), turning it into the first-order equation:

$$2k_\mu \mathbf{A}^\mu \times \mathbf{A}^{\nu\prime} - k^\nu \mathbf{A}_\mu \times \mathbf{A}^{\mu\prime} + \mathbf{A}_\mu \times (\mathbf{A}^\mu \times \mathbf{A}^\nu) = 0. \tag{105}$$

The gauge condition (56), imposed on 4-potential divergence, means that

$$k_\mu \mathbf{A}^\mu = \mathbf{q} = \text{const} = 0. \tag{106}$$

As in other wave problems considered above, we suppose that vector $\mathbf{q}$ (106) is vanishing. Availability of nonzero $\mathbf{q}$ would correspond to the wave propagation on the background of some external stationary field.

With regard to (106), the first term in the wave equation (105) vanishes, and the wave equation takes the form:

$$\overset{abc}{\varepsilon}\ k^\nu \overset{b}{\mathrm{A}}_\mu \overset{c}{\mathrm{A}}{}^{\mu\prime} + \overset{ab}{\mathrm{I}}\ \overset{b}{\mathrm{A}}{}^\nu = 0. \tag{107}$$

where $\overset{ab}{\mathrm{I}}$ is Yang-Mills inertia tensor, introduced above.

The twelve equations (107) are not independent from each other. The scalar multiplication (107) by the wave vector k^ν gives three identities of the form $0 = 0$. The scalar multiplication (107) by the potential $\mathbf{A}^\nu$ generates the algebraic identity, which should be followed by the solution (107):

$$\mathfrak{U} = \overset{ab}{\mathrm{S}}\ \overset{ab}{\mathrm{S}} - \left(\overset{bb}{\mathrm{S}}\right)^2 = 0. \tag{108}$$

The expression (108) is a certain "pale shadow" of the energy conservation law for ymiton. For the degenerate wave (107) it is impossible to introduce anything resembling the kinetic energy $\mathfrak{K}$. The analogue of potential energy, determined by formula (108), is identically zero.

The system, in which

$$k^\nu = \{1;\ 1;\ 0;\ 0\},$$

will be named the ymiton "intrinsic frame of reference".

This is a "frame of unit frequency", in which axis x is directed along the wave direction. In this frame $\phi = t - x$. Using the notation of the form "1+3" for potential $\overset{a}{\mathrm{A}}{}^\nu$ again:

$$\overset{a}{\mathrm{A}}{}^\nu = \{\overset{a}{\mathrm{T}};\ \overset{a}{\mathbf{U}}\},$$

we can reduce the ymiton wave problem to the following form:

$$\overset{a}{\mathrm{T}}=\overset{a}{\mathrm{U}}_1; \tag{109}$$

$$\overset{ab}{\mathrm{I}}\,\overset{b}{\mathrm{U}}_2=0; \tag{110}$$

$$\overset{ab}{\mathrm{I}}\,\overset{b}{\mathrm{U}}_3=0; \tag{111}$$

$$\overset{ab}{\mathrm{I}}\,\overset{b}{\mathrm{U}}_1=\overset{a}{\mathrm{g}}\,. \tag{112}$$

The equations (109) allow to exclude time components $\overset{a}{\mathrm{T}}$ of all the three potentials of YM-triplet from consideration.

YM-vector **g**, entering into (112), similarly to the components of YM-inertia tensor $\overset{ab}{\mathrm{I}}$, is expressed by spatial triplet components $\overset{a}{\mathrm{U}}_i$ $\left(i=\overline{2,\,3}\right)$:

$$\overset{1}{\mathrm{g}}=\overset{3}{\mathrm{U}}_2\,\overset{2}{\mathrm{U}}_2{}'+\overset{3}{\mathrm{U}}_3\,\overset{2}{\mathrm{U}}_3{}'-\left(\overset{2}{\mathrm{U}}_2\,\overset{3}{\mathrm{U}}_2{}'+\overset{2}{\mathrm{U}}_3\,\overset{3}{\mathrm{U}}_3{}'\right);$$
$$\overset{2}{\mathrm{g}}=\overset{1}{\mathrm{U}}_2\,\overset{3}{\mathrm{U}}_2{}'+\overset{1}{\mathrm{U}}_3\,\overset{3}{\mathrm{U}}_3{}'-\left(\overset{3}{\mathrm{U}}_2\,\overset{1}{\mathrm{U}}_2{}'+\overset{3}{\mathrm{U}}_3\,\overset{1}{\mathrm{U}}_3{}'\right);$$
$$\overset{3}{\mathrm{g}}=\overset{2}{\mathrm{U}}_2\,\overset{1}{\mathrm{U}}_2{}'+\overset{2}{\mathrm{U}}_3\,\overset{1}{\mathrm{U}}_3{}'-\left(\overset{1}{\mathrm{U}}_2\,\overset{2}{\mathrm{U}}_2{}'+\overset{1}{\mathrm{U}}_3\,\overset{2}{\mathrm{U}}_3{}'\right).$$

Yang-Mills inertia tensor $\overset{ab}{\mathrm{I}}$ for the ymiton wave problem looks as follows:

$$\overset{11}{\mathrm{I}}=\left(\overset{2}{\mathrm{U}}_2\right)^2+\left(\overset{2}{\mathrm{U}}_3\right)^2+\left(\overset{3}{\mathrm{U}}_2\right)^2+\left(\overset{3}{\mathrm{U}}_3\right)^2;$$
$$\overset{22}{\mathrm{I}}=\left(\overset{3}{\mathrm{U}}_2\right)^2+\left(\overset{3}{\mathrm{U}}_3\right)^2+\left(\overset{1}{\mathrm{U}}_2\right)^2+\left(\overset{1}{\mathrm{U}}_3\right)^2;$$
$$\overset{33}{\mathrm{I}}=\left(\overset{1}{\mathrm{U}}_2\right)^2+\left(\overset{1}{\mathrm{U}}_3\right)^2+\left(\overset{2}{\mathrm{U}}_2\right)^2+\left(\overset{2}{\mathrm{U}}_3\right)^2;$$
$$\overset{12}{\mathrm{I}}=\overset{21}{\mathrm{I}}=-\left(\overset{1}{\mathrm{U}}_2\,\overset{2}{\mathrm{U}}_2+\overset{1}{\mathrm{U}}_3\,\overset{2}{\mathrm{U}}_3\right);$$
$$\overset{13}{\mathrm{I}}=\overset{31}{\mathrm{I}}=-\left(\overset{1}{\mathrm{U}}_2\,\overset{3}{\mathrm{U}}_2+\overset{1}{\mathrm{U}}_3\,\overset{3}{\mathrm{U}}_3\right);$$
$$\overset{23}{\mathrm{I}}=\overset{32}{\mathrm{I}}=-\left(\overset{2}{\mathrm{U}}_2\,\overset{3}{\mathrm{U}}_2+\overset{2}{\mathrm{U}}_3\,\overset{3}{\mathrm{U}}_3\right).$$

The expression for "potential energy" $\mathfrak{U}$ (108) is reduced to the form which contains the sum of three squares:

$$\mathfrak{U}=\mathrm{p}_1^2+\mathrm{p}_2^2+\mathrm{p}_3^2,$$

where

$$\mathrm{p}_1=\overset{1}{\mathrm{U}}_2\,\overset{2}{\mathrm{U}}_3-\overset{2}{\mathrm{U}}_2\,\overset{1}{\mathrm{U}}_3;$$
$$\mathrm{p}_2=\overset{1}{\mathrm{U}}_2\,\overset{3}{\mathrm{U}}_3-\overset{1}{\mathrm{U}}_3\,\overset{3}{\mathrm{U}}_2;$$
$$\mathrm{p}_3=\overset{2}{\mathrm{U}}_2\,\overset{3}{\mathrm{U}}_3-\overset{2}{\mathrm{U}}_3\,\overset{3}{\mathrm{U}}_2\,.$$

The equality $\mathfrak{U} = 0$ forms three algebraic conditions, imposed on the spatial components of the potential:

$$\begin{aligned} \mathrm{p}_1 &= 0; \\ \mathrm{p}_2 &= 0; \\ \mathrm{p}_3 &= 0. \end{aligned} \tag{113}$$

The relations, provided here, allow formulating the following algorithm of the construction of ymiton wave parameters:

(1) The equations (110) and (113) give, with regard to the expressions for $\overset{ab}{\mathrm{I}}$ and p_i, the system of six algebraic equations for determination of "six-vector" $\overset{a}{\mathrm{U}}_i$, ($a = 1,2,3$; $i = 2,3$) three homogeneous cubic equations (110) and three homogeneous equations of the second-order (113). Let us assume that there is at least one solution to this algebraic system. Apparently, this solution will also satisfy the equations (111) which result from (110) by substitution of the Cartesian subscripts $2 \leftrightarrow 3$ in the equations (110) and in tensor $\overset{ab}{\mathrm{I}}$ which is invariant relative to such substitution.
Due to homogeneity of the equations (110) and (113), if $\overset{a}{\mathrm{U}}_i = \overset{a}{\kappa}_i$ is some solution to these equations, the expression

$$\overset{a}{\mathrm{U}}_i = \mathrm{f}(\phi)\ \overset{a}{\kappa}_i \quad (a = 1,2,3;\ i = 2,3)\,.$$

is also a solution to this system under the arbitrary function $\mathrm{f}(\phi)$, which depends on the wave phase, but does not depend on Yang-Mills and Cartesian indices. It goes without saying that it is worth speaking about a "wave" only in case if $\mathrm{f}(\phi)$ is a bounded oscillatory function.

(2) By substituting the relation for $\overset{a}{\mathrm{U}}_i$ into the expressions which determine YM-vector **g** and matrix $\overset{ab}{\mathrm{I}}$, we solve the equations (112) as a system of three linear heterogeneous algebraic equations relative to the longitudinal components of potentials $\overset{a}{\mathrm{U}}_1$.
It is easy to see that the components of the inertia tensor $\overset{ab}{\mathrm{I}}$ are proportional to the square of arbitrary function $\mathrm{f}(\phi)$, and the components of vector **g** are proportional to the product of $\mathrm{f}(\phi) \cdot \mathrm{f}'(\phi)$. Therefore, the dependence of the longitudinal components of the potential on the wave phase ϕ has the following form:

$$\overset{a}{\mathrm{U}}_1 \approx \frac{\mathrm{f}'}{\mathrm{f}},$$

which inevitably generates singularities in the longitudinal components $\overset{a}{\mathrm{U}}_1$ with the continuous function $\mathrm{f}(\phi)$ oscillating around zero.
The condition of chiral determinacy for this wave, in forms (86) and (87), imposes restriction on the behavior of $\mathrm{f}(\phi)$.
This restriction requires a constant sign of product $\mathrm{f} \cdot \mathrm{f}'$, which, actually, requires discontinuous oscillations of function $\mathrm{f}(\phi)$.

On the whole, the provided description of "Yang-Mills photon" looks rather confusing. Unlike tymos wave ($k^\nu k_\nu > 0$) and terriculum wave ($k^\nu k_\nu < 0$), isotropic YM-ymiton wave is an "algebraic object" which does not require differential wave equations for its description.

9.2.14 About Free Zero-current Triplet Field Statistics

In the preceding parts of this article (9.2.10-9.2.13) some certain difficulties in the description of Yang-Mills waves in the wave cone $k^\nu k_\nu \leq 0$ were mentioned. These difficulties include unbounded growth of potentials in the terriculum wave – if the procedure of chiralization is not used, and the discontinuous behavior of waves ymiton. These difficulties may indicate that the problem of describing the free wave state of Yang-Mills triplet is not a correctly-set problem in theoretical physics. The "free field" can probably turn into a "non-free" field in space-like cone $k^\nu k_\nu < 0$ of the four-dimensional space of the wave vectors, generating YM-currents. In the linear singlet sector of physics there is nothing similar to this phenomenon.
But, at the very least, the description of the free triplet waves in time-like cone $k^\nu k_\nu > 0$ is quite correct mathematically - no matter whether we use the chaotic chiral-symmetric solutions of the wave equations, or impose chiral determinacy to a solution by using the procedure of "soft chiralization".
Consequently, we can formulate a classical problem of the statistical characteristics of the free triplet field in thermodynamic equilibrium - the problem, similar to the one that Max Planck successfully solved in 1900 for maxwellian singlet by constructing the frequency energy distribution function. Unlike the one-dimensional Planck distribution, the corresponding Yang-Mills distribution must be two-dimensional, due to the absence of constraint between frequency ω and three-dimensional wave vector $|\mathbf{k}|$ in the triplet sector.
It can be supposed that the study of the thermodynamics of the triplet field is an unconventional problem. The language of Fourier expansion of the triplet field into plane waves may not appear to be very convenient language for the nonlinear theory.
"Photon" in the electromagnetic theory retains its identity from one process of interaction with matter (emission or scattering) to the other one (absorption or scattering). Due to the lack of dispersion, wave packets in the electromagnetic theory spread without changing the profile, while retaining its identity. All these identities make Fourier language useful for the singlet sector of physics. All these identities are not retained in Yang-Mills nonlinear theory of the triplet waves. Yang-Mills wave packets spread and interact with each other, changing the profiles and absorbing other packets. Perhaps, the thermodynamics of Yang-Mills fields requires some other variables for its description.

9.3 One-current Triplet States

9.3.1 Classification and Lagrangians of One-current Triplet States

According to the scheme of the pure states, demonstrated in figure 1, and the structure of the weak Lagrangian L_w (49), we can describe all the possible pure triplet one-current states in the following way.

1. There are two similar "heavy" current states with a space-like current vector. The Lagrangian of these states, according to (49), can be formulated in the following way:

$$L_w = \frac{1}{2}\,\mathrm{j}^\nu\,\mathrm{j}_\nu - \mathrm{j}^\nu\,\overset{k}{\mathrm{A}}_\nu - \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot\mathbf{A}_{\mu\nu}, \tag{114}$$

where $\mathrm{j}^\nu = \frac{1}{2}\,\overset{k}{\mathrm{j}}{}^\nu$; $k = 1$ or $k = 2$.
In these states only one of the three YM-currents with YM-number equal to 1 or 2, is nonzero.
There are three types of states, described by the Lagrangian (114):

- "Heavy" wave states, in which the current vector is nonzero all over the 4-dimensional space (there is no one pomerium).
- Stationary states, in which there is one stationary pomerium in a certain intrinsic system of states the current zone boundaries in which current isotropization j^ν ($\mathrm{j}^\nu\,\mathrm{j}_\nu = 0$) takes place; there is no current beyond this boundary; inside the current zone there are cavitated zones (latebrae) inside which there is no current; on the boundaries of these cavitated tubes, the pseudo-Euclidean square of 4-current has the maximum possible value (7).
  While solving the problems of stationary one-current states, it is worth taking into account the curved space-time, generated by a high density of energy-momentum inside the current zone and around it. As well as quarks, the stationary one-current states of the triplet sector (in contrast to leptons – the stationary states of the singlet sector) may turn out to be physically unrealizable due to the "wrong behavior" of stationary potential triplet $\mathbf{A}^\nu$ away from the current zone, resulting in divergence of the total energy of the stationary state.
- Non-stationary states, or the "interaction states", in which there is one, or more than one, non-stationary boundary of the current zones[31].

Varying the action functional with the Lagrangian (114) by the triple of potentials $\mathbf{A}^\nu$ and by current j^ν, we obtain the following field equations for a "heavy" one-current triplet state:

$$\partial_\mu\mathbf{A}^{\mu\nu} + \mathbf{A}_\mu \times \mathbf{A}^{\mu\nu} = 4\pi\,\mathrm{j}^\nu\,\overset{k}{\mathbf{e}}, \tag{115}$$

$$\mathrm{j}^\nu + \overset{k}{\mathrm{A}}{}^\nu = 0. \tag{116}$$

In Yang-Mills equations (115) $\overset{k}{\mathbf{e}}$ is a Yang-Mills unit vector of k YM-direction, i.e. YM-vector with YM-components $\{1;\,0;\,0\}$ for $k = 1$ and $\{0;\,1;\,0\}$ for $k = 2$.
The second field equation, connecting the current and potential, is true in the current zone, where $\mathrm{j} \neq 0$.
One gauge condition can be imposed on the solution of Yang-Mills equations. As in the problem on Yang-Mills free fields, it is convenient to apply potential 4-divergence gauge:

$$\partial_\nu\mathbf{A}^\nu = 0. \tag{117}$$

[31] Currently having no numerical solutions to stationary problems and having no version of the existence theorem which would guarantee the existence of solutions to stationary problems, we are not going to deal with formulation of non-stationary problems in this article.

The differential condition for triplet currents (19), which, by means of the symbols applied here, can be written in the form:

$$\overset{k}{\mathbf{e}}\,\partial_\nu \mathrm{j}^\nu + \mathrm{j}^\nu \mathbf{A}_\nu \times \overset{k}{\mathbf{e}}{}^\nu = 0, \tag{118}$$

gives the following relation in YM-projections:

$$\partial_\nu \mathrm{j}^\nu = 0, \tag{119}$$

$$\overset{3}{\mathrm{A}}_\nu\, \mathrm{j}^\nu = 0,\ \ \overset{\not k}{\mathrm{A}}_\nu\, \mathrm{j}^\nu = 0, \tag{120}$$

where $\not k = 1$ with $k = 2$; $\not k = 2$ with $k = 1$.

The condition of current conservation j^ν (119) is compatible with gauge (117) and the field equation (116).

The orthogonality conditions (120) are the result of differential condition for currents (118), which, in its turn, is the result of the field equations (115) and (116). Therefore, the orthogonality conditions (120) should not be included (together with undetermined Lagrange's multipliers) into the Lagrangian expansion of one-current problem (114).

The following dilemma can be formulated: either there are solutions to field equations (115), (116), (117), (119), compatible with the orthogonality conditions (120) at zero Lagrange's multipliers, or one-current triplet states should not be treated as real physical states at all – they exist only on the background of some external non-physical "Higgs-like" field which is mathematically reflected in the Lagrange's multipliers for the extended Lagrangian.

Getting rid of the weak field tensor $\mathbf{A}^{\mu\nu}$ in the equations (115) and considering the gauge (117), we can formulate the Yang-Mills equations of one-current problem in the following way:

$$-\Box \mathbf{A}^\nu + 2\mathbf{A}_\mu \times \partial^\mu \mathbf{A}^\nu - \mathbf{A}_\mu \times \partial^\nu \mathbf{A}^\mu + \hat{\mathrm{I}}\mathbf{A}^\nu = 4\pi\, \mathrm{j}^\nu\, \overset{k}{\mathbf{e}}, \tag{121}$$

where YM-inertia tensor $\hat{\mathrm{I}}$, according to (75) and (77) has the following YM-components

$$\overset{ab}{\mathrm{I}} = \overset{a}{\mathrm{A}}{}^\mu\ \overset{b}{\mathrm{A}}_\mu - \overset{ab}{\delta}\ \overset{c}{\mathrm{A}}_\mu\, \overset{c}{\mathrm{A}}{}^\mu, \tag{122}$$

2. Besides the two "heavy" one-current triplet states, there are three similar neutrino states, with isotropic current vector N^ν ($N^\nu N_\nu = 0$). The Lagrangian of these neutrino YM-states, which were named *ymino* above, looks as follows:

$$L_{w,N} = -N^\nu\ \overset{k}{\mathrm{A}}_\nu - \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot \mathbf{A}_{\mu\nu} - \frac{1}{2}\lambda N^\nu N_\nu, \tag{123}$$

($k = 1,\ 2,\ 3$).

The last term in (123) is the "penalty" for current isotropy condition, λ is a Lagrange's multiplier. The current N^ν in (123) corresponds to a nonzero k – YM-component of YM-current triplet. With $k = 1$ or $k = 2$ along with the neutrino states (123), there are current states with the Lagrangian (114). If $k = 3$, there is only neutrino state with the Lagrangian (123).

The field equations, following from the Lagrangian (123), look similar to the equations (115), (116) and (121):

$$\partial_\mu \mathbf{A}^{\mu\nu} + \mathbf{A}_\mu \times \mathbf{A}^{\mu\nu} = 4\pi\, N^\nu \overset{k}{\mathbf{e}}, \tag{124}$$

$$-\Box \mathbf{A}^\nu + 2\mathbf{A}_\mu \times \partial^\mu \mathbf{A}^\nu - \mathbf{A}_\mu \times \partial^\nu \mathbf{A}^\mu + \hat{\mathrm{I}}\mathbf{A}^\nu = 4\pi\, N^\nu \overset{k}{\mathbf{e}}, \tag{125}$$

$$\lambda N^\nu + \overset{k}{\mathrm{A}}{}^\nu = 0, \tag{126}$$

$$\partial_\nu \mathbf{A}^\nu = 0, \tag{127}$$

$$\partial_\nu N^\nu = 0, \tag{128}$$

$$N^\nu \overset{\not k}{\mathrm{A}}_\nu = 0, \tag{129}$$

$$N^\nu N_\nu = 0. \tag{130}$$

In orthogonality conditions (129) $\not k = 2$ and 3 with $k = 1$; $\not k = 3$ and 1 with $k = 2$; $\not k = 1$ and 2 with $k = 3$.
From field equation (126) and the condition of neutrino current isotropy (130) there follows the orthogonality condition

$$N^\nu \overset{k}{\mathrm{A}}_\nu = 0. \tag{131}$$

Condition (131), combined with condition (129), means that neutrino current N^ν must be orthogonal to all the three triplet potentials. Besides this, as it follows from (126), potential $\overset{k}{\mathrm{A}}{}^\nu$ is an isotropic vector.
Differentiating the field equation (126) by coordinates x_ν, and taking into account (127) and (128), we find that 4-gradient of the Lagrange's multiplier λ satisfies the orthogonality condition:

$$N^\nu \partial_\nu \lambda = 0. \tag{132}$$

This condition will be anyway satisfied, if $\lambda = \mathrm{const}$.

9.3.2 "Heavy" One-current Plane Triplet Wave

We will try the solution to Yang-Mills wave equations (121) in the form of a plane wave, supposing that current j^ν and potential $\mathbf{A}^\nu$ depend on the only argument – the wave phase $\phi = k^\mu x_\mu$. The wave equation (121) for a plane wave gets the following form

$$k^\mu k_\mu \mathbf{A}^{\nu\,\prime\prime} + 2k^\mu \mathbf{A}_\mu \times \mathbf{A}^{\nu\,\prime} - k^\nu \mathbf{A}_\mu \times \mathbf{A}^{\mu\,\prime} + \hat{\mathrm{I}}\mathbf{A}^\nu + 4\pi\ \overset{k}{\mathrm{A}}{}^\nu\ \overset{k}{\mathbf{e}} = 0. \tag{133}$$

In equation (133) the prime denotes a wave phase derivative. In the last term, field equation (116) is taken into account. The summation by "stationary" index k is not made in this term.
If homogeneous equations (133) are solved, the current is calculated by field equation (116).
Gauge condition (117) in the plane wave is reduced to the condition:

$$k_\nu \mathbf{A}^{\nu\,\prime} = 0,$$

with integral:

$$k_\nu \mathbf{A}^\nu = \mathbf{q} = \text{const},$$

but, as in the other wave problems, we will assume that $\mathbf{q} = 0$: nonzeroness of $\mathbf{q}$ would correspond to a wave propagation on the background of a certain external stationary field. Considering zero value of $\mathbf{q}$, the second term in the equation (133) vanishes:

$$k^\mu k_\mu \mathbf{A}^{\nu\,\prime\prime} - k^\nu \mathbf{A}_\mu \times \mathbf{A}^{\mu\,\prime} + \hat{\mathrm{I}}\mathbf{A}^\nu + 4\pi \overset{k}{\mathrm{A}}{}^\nu \overset{k}{\mathbf{e}} = 0. \tag{134}$$

By scalar multiplying the plane one-current wave equation (134) by wave vector k^ν, we find with anisotropic vector k^ν $(k^\nu k_\nu \neq 0)$ that:

$$\mathbf{A}_\mu \times \mathbf{A}^{\mu\,\prime} = 0. \tag{135}$$

As in Yang-Mills free wave problem, relation (135) can be considered a non-holonomic constraint, imposed on the initial values of potentials and their derivatives. Considering condition (135), the wave equation (134) takes even a simpler form:

$$k^\mu k_\mu \mathbf{A}^{\nu\,\prime\prime} + \hat{\mathrm{I}}\mathbf{A}^\nu + 4\pi \overset{k}{\mathrm{A}}{}^\nu \overset{k}{\mathbf{e}} = 0. \tag{136}$$

Let ε be a non-Euclidean module of wave vector k^ν:

$$k^\nu k_\nu = \pm\varepsilon^2,$$

where a plus sign corresponds to the wave with a time-like wave vector, but a minus sign corresponds to the wave with a space-like wave vector. Making a scale transformation in (136),

$$k^\nu \to \varepsilon k^\nu;\ \phi \to \varepsilon\phi,$$

normalizing wave vector k for a unit module, we get rid of the module of wave vector ε in equation (136):

$$\pm\mathbf{A}^{\nu\,\bullet\bullet} + \hat{\mathrm{I}}\mathbf{A}^\nu + 4\pi \overset{k}{\mathrm{A}}{}^\nu \overset{k}{\mathbf{e}} = 0. \tag{137}$$

A plus sing in (137) corresponds to the wave with time-like wave vector k^ν, normalized for unit:

$$k^\nu k_\nu = 1.$$

A minus sign in (137) corresponds to the wave with space-like wave vector k^ν, normalized for unit:

$$k^\nu k_\nu = -1.$$

The dots on the right-side of the potential symbol imply the derivative with respect to ε - rescaled wave phase. In the wave intrinsic frame of reference, wave vector k^ν has the form:

$$k^\mu = \{1;\ 0;\ 0;\ 0\}-$$

for time-like vector k^ν and

$$k^\mu = \{0;\ 1;\ 0;\ 0\}-$$

for space-like vector k^ν.
In the wave intrinsic frame of reference the phase derivative coincides with the time derivative (all wave characteristics in the intrinsic frame depend only on time) – for the time-like wave vector, or with longitudinal coordinate x derivatives – for the wave with a space-like wave vector (in such wave all the characteristics depend only on longitudinal coordinate; in the intrinsic frame of reference such a wave is a kind of ripple of current and potentials stiffed in space).
Multiplying one-current plane wave equation (137) by $\mathbf{A}^{\nu\,\bullet}$ and integrating by the wave phase, we can obtain the energy integral for this wave:

$$\mathfrak{K} + \mathfrak{U} = \mathrm{E} = \mathrm{const},$$

where

$$\mathfrak{U} = \frac{1}{4}\left(\left(\overset{aa}{\mathrm{S}}\right)^2 - \overset{ab}{\mathrm{S}}\overset{ab}{\mathrm{S}} + 8\pi \overset{k}{\mathrm{A}}{}^{\nu} \overset{k}{\mathrm{A}}_{\nu}\right) -$$

is a wave potential energy, and

$$\overset{ab}{\mathrm{S}} = \overset{a}{\mathrm{A}}{}^{\mu} \overset{b}{\mathrm{A}}_{\mu}\ .$$

In the expression for potential energy $\mathfrak{U}$, the summation is made by YM-repetitive indices a, b. But the summation by the "stationary" YM-index k is missing.
"Kinetic" energy $\mathfrak{K}$ for the waves with space-like and time-like wave vector differs in sign:
$\mathfrak{K} = \mathfrak{K}_t = -\frac{1}{2}\mathbf{A}^{\nu\,\bullet} \cdot \mathbf{A}^{\bullet}_{\nu}$, (time-like wave),
$\mathfrak{K} = \mathfrak{K}_s = \frac{1}{2}\mathbf{A}^{\nu\,\bullet} \cdot \mathbf{A}^{\bullet}_{\nu}$, (space-like wave).
For time-like wave, energy E is positive, but for a space-like wave the energy sign E can be either positive, or negative.
In the expression for potential energy $\mathfrak{U}$ the first two terms have the fourth order by potentials $\overset{a}{\mathrm{A}}_{\mu}$, and the last term, generated by current availability in the wave, is quadratic by potentials.
Availability of this term destroys the self-similarity of a plane wave relative to the energy that is inherent to a zero-current wave. For a one-current wave, energy E is a significant parameter (not just the scale one).
Equations (137) describe some dynamical system. By analogy with dynamical system (79), which describes a zero-current wave, let us call dynamical system (137) with time-like vector k^ν a "1c-tymos" (one-current-tripletic-yang-mills' oscillator). The choice of a plus sign before the first term of the equation corresponds to 1-tymos object (137). The choice of a minus sign before this term, corresponding to space-like wave-vector k^ν, generates a one-current analogue of the "terriculum" object. Let us name this object a 1c-terriculum.

9.3.3 Detailed Notation of the 1-tymos Dynamics Equations

Let us use "1+3" – the notations for the components of weak potential $\overset{a}{\mathrm{A}}{}^{\nu}$ (70). In these notations, it follows from the condition (117) that in the intrinsic frame of reference of

1-tymos wave all the time components of the potential are vanishing:

$$\overset{a}{\mathrm{T}}= 0,\ \ a = \overline{1,\,3}.$$

By choosing "fixed" YM-index k equal to one, for definiteness, we can write the equations of a plane wave (137) as a system of equations for the three three-dimensional vectors $\overset{a}{\mathbf{U}}$ which specify the spatial components of YM-potentials

$$\overset{a}{\mathbf{U}}{}^{\bullet\bullet} + \overset{ab}{\mathrm{I}}\,\overset{b}{\mathbf{U}} + 4\pi\ \overset{1}{\mathbf{U}} \cdot \overset{1a}{\delta} = 0, \tag{138}$$

where tensor $\overset{ab}{\mathrm{I}}$ has the same form (78), (83) as for the zero-current wave. If in system (138) we explicitly write out the YM-indices of all vectors $\overset{a}{\mathbf{U}}$ and all their Cartesian indices, we will get a system of nine differential equations, in which only the first three equations with a unit YM-index are different from the analogous system for a zero-current wave (82):

$$\begin{aligned}
&\overset{1}{\mathrm{U}}_1{}^{\bullet\bullet} + \left(\overset{11}{\mathrm{I}} + 4\pi\right)\overset{1}{\mathrm{U}}_1 + \overset{12}{\mathrm{I}}\,\overset{2}{\mathrm{U}}_1 + \overset{13}{\mathrm{I}}\,\overset{3}{\mathrm{U}}_1 = 0,\\
&\overset{1}{\mathrm{U}}_2{}^{\bullet\bullet} + \left(\overset{11}{\mathrm{I}} + 4\pi\right)\overset{1}{\mathrm{U}}_2 + \overset{12}{\mathrm{I}}\,\overset{2}{\mathrm{U}}_2 + \overset{13}{\mathrm{I}}\,\overset{3}{\mathrm{U}}_2 = 0,\\
&\overset{1}{\mathrm{U}}_3{}^{\bullet\bullet} + \left(\overset{11}{\mathrm{I}} + 4\pi\right)\overset{1}{\mathrm{U}}_3 + \overset{12}{\mathrm{I}}\,\overset{2}{\mathrm{U}}_3 + \overset{13}{\mathrm{I}}\,\overset{3}{\mathrm{U}}_3 = 0,
\end{aligned} \tag{139}$$

but the other six equations for YM-indices 2 and 3 coincide with the corresponding equations (82).

Energy integral for system (138) has the form which is analogous to the energy integral for zero-current wave (81):

$$\frac{1}{2}\,\overset{a}{\mathbf{U}}{}^{\bullet}\cdot\overset{a}{\mathbf{U}}{}^{\bullet} + \frac{1}{2}\left(\left(\overset{1}{\mathbf{U}}\times\overset{2}{\mathbf{U}}\right) + \left(\overset{2}{\mathbf{U}}\times\overset{3}{\mathbf{U}}\right) + \left(\overset{3}{\mathbf{U}}\times\overset{1}{\mathbf{U}}\right) + 4\pi\,\overset{1}{\mathbf{U}}{}^2\right) = \mathrm{E}.$$

In contrast to a zero-current wave problem, the 1c-wave cannot be "scaled" by energy E. The equations of non-holonomic constraints (135) in this problem have the same form (85) as in the problem on a zero-current wave. Chiral determinacy conditions (87) also coincide in these problems.

As in the case of a zero-current wave, the one-current wave problem takes especially simple form for the "orthogonal" wave, in which

$$\overset{a}{\mathrm{U}}_i = 0,\ \text{with}\, i \neq a.$$

For such wave the equations of non-holonomic constraints are satisfied identically, and for the three nontrivial potential components $\overset{a}{\mathrm{U}}_i$ with $i = a$, it is possible to formulate the system of equations, similar to the system (90):

$$\begin{aligned}
&\mathrm{U}_1^{\bullet\bullet} + \mathrm{U}_1\left(4\pi + \mathrm{U}_2^2 + \mathrm{U}_3^2\right) = 0,\\
&\mathrm{U}_2^{\bullet\bullet} + \mathrm{U}_2\left(\mathrm{U}_3^2 + \mathrm{U}_1^2\right) = 0,\\
&\mathrm{U}_3^{\bullet\bullet} + \mathrm{U}_3\left(\mathrm{U}_1^2 + \mathrm{U}_2^2\right) = 0.
\end{aligned} \tag{140}$$

Energy integral for (140) looks as follows:

$$\frac{1}{2}\left(\overset{\bullet}{\mathrm{U}}_1{}^2+\overset{\bullet}{\mathrm{U}}_2{}^2+\overset{\bullet}{\mathrm{U}}_3{}^2\right)+\frac{1}{2}\left(\mathrm{U}_1^2\mathrm{U}_2^2+\mathrm{U}_2^2\mathrm{U}_3^2+\mathrm{U}_3^2\mathrm{U}_1^2+4\pi\,\mathrm{U}_1^2\right)=\mathrm{E}.^{32} \tag{141}$$

The chiral determinacy condition for the solution (140) has a simple form:

$$\mathrm{U}_1\mathrm{U}_2\mathrm{U}_3 \geq 0. \tag{142}$$

The procedure of "soft chiralization" for (140) includes the sign reverse for derivatives $\overset{\bullet}{\mathrm{U}}_i$ $\left(i=\overline{1,\,3}\right)$ at vanishing of any potential component U_i.
Probably, with higher energy of E the one-current wave behaves like a zero-current wave: in equations (140) constant terms 4π, compared to the large (average) squares of potentials, can be neglected. At low oscillation amplitude of potentials, the first equation (140) turns into harmonic oscillator equation, and the second and third equations are transformed into Mathieu equations. In the situation with small-amplitudes the three-component system (140) is similar to the behavior described in the article [1] maxwellian "heavy photon", but modulated by the energy exchange "heavy photon" U_1 with two other YM-degrees of freedom U_2 and U_3 in the mode as it is dictated by the Mathieu equation. In particular, without taking into account the condition of the chiral determinacy (142), system (140) may have periodic solutions.

9.3.4 Dynamics of a One-current-terriculum

Repeating the calculations, described in p.9.2.10 for zero-current terriculum, as applied to the problem of a one-current wave with space-like wave vector, we find that all of the longitudinal components of the potentials are equal to zero: $\overset{a}{\mathrm{U}}_1=0$, and nine non-trivial components $\overset{a}{\mathrm{T}}$, $\overset{a}{\mathrm{U}}_i$ with $i=2,\,3$ satisfy the system of equations similar to the system of equations (96), which describes zero-current terriculum. In the equations of a one-current wave only the three first equations, corresponding to YM-index 1, differ from the system of equations (96). These equations take the form:

$$\begin{aligned}
&-\overset{1}{\mathrm{T}}{}^{\bullet\bullet}+\left(\overset{11}{\mathrm{I}}+4\pi\right)\overset{1}{\mathrm{T}}+\overset{12}{\mathrm{I}}\overset{2}{\mathrm{T}}+\overset{13}{\mathrm{I}}\overset{3}{\mathrm{T}}=0,\\
&-\overset{1}{\mathrm{U}}_2{}^{\bullet\bullet}+\left(\overset{11}{\mathrm{I}}+4\pi\right)\overset{1}{\mathrm{U}}_2+\overset{12}{\mathrm{I}}\overset{2}{\mathrm{U}}_2+\overset{13}{\mathrm{I}}\overset{3}{\mathrm{U}}_2=0,\\
&-\overset{1}{\mathrm{U}}_3{}^{\bullet\bullet}+\left(\overset{11}{\mathrm{I}}+4\pi\right)\overset{1}{\mathrm{U}}_3+\overset{12}{\mathrm{I}}\overset{2}{\mathrm{U}}_3+\overset{13}{\mathrm{I}}\overset{3}{\mathrm{U}}_3=0.
\end{aligned} \tag{143}$$

The other six equations of the system (96) are transferred into the problem of a one-current wave terriculum unchanged. No changes take place while the components of tensor $\overset{ab}{\mathrm{I}}$ (97) and the equations of nonholonomic constraints (98) are transferred into this problem. As in case of the zero-current terriculum problem, the energy integral of a

[32] It can be noted that the "energy" of oscillator (140), determined by the formula (32), differs from the wave energy density in the intrinsic frame of reference in multiplier 4π.

one-current terriculum is proper to formulate by using three complex potentials $\overset{a}{\boldsymbol{\tau}}$ (100). When using vectors $\overset{a}{\boldsymbol{\tau}}$, the energy integral for (143) takes the following form:

$$
\begin{aligned}
&\mathfrak{K} + \mathfrak{U} = \mathrm{E},\\
&\mathfrak{K} = \frac{1}{2}\left(\left(\overset{1}{\boldsymbol{\tau}}{}^{\bullet}\right)^2 + \left(\overset{2}{\boldsymbol{\tau}}{}^{\bullet}\right)^2 + \left(\overset{3}{\boldsymbol{\tau}}{}^{\bullet}\right)^2\right),\\
&\mathfrak{U} = \frac{1}{2}\left(\left(\overset{1}{\boldsymbol{\tau}} \times \overset{2}{\boldsymbol{\tau}}\right)^2 + \left(\overset{2}{\boldsymbol{\tau}} \times \overset{3}{\boldsymbol{\tau}}\right)^2 + \left(\overset{3}{\boldsymbol{\tau}} \times \overset{1}{\boldsymbol{\tau}}\right)^2 + 4\pi\left(\overset{1}{\boldsymbol{\tau}}\right)^2\right).
\end{aligned}
$$

Chiral determinant CD for this problem is defined by the same formula (101) as for the zero-current problem. When CD is vanishing, all derivatives $\overset{a}{\boldsymbol{\tau}}{}^{\bullet}$ signs must be reversed. If we do not appeal to the chiralization procedure, the potential components in one-current wave terriculum grow indefinitely along the longitudinal coordinates of the wave.
The problem of 1c-terriculum object takes particularly simple form for the "orthogonal wave", in which

$$
\begin{aligned}
&\overset{1}{\boldsymbol{\tau}} = \{\mathrm{T};\ 0;\ 0\},\\
&\overset{2}{\boldsymbol{\tau}} = \{0;\ i\mathrm{U}_2;\ 0\},\\
&\overset{3}{\boldsymbol{\tau}} = \{0;\ 0;\ i\mathrm{U}_3\}.
\end{aligned}
$$

For this three-component wave the equations of nonholonomic constraints are satisfied identically. The wave equations take the form:

$$
\begin{aligned}
&-\mathrm{T}^{\bullet\bullet} + \mathrm{E}\left(4\pi + \mathrm{U}_2^2 + \mathrm{U}_3^2\right) = 0,\\
&-\mathrm{U}_2^{\bullet\bullet} + \mathrm{U}_2\left(\mathrm{U}_3^2 - \mathrm{T}^2\right) = 0,\\
&-\mathrm{U}_3^{\bullet\bullet} + \mathrm{U}_3\left(\mathrm{U}_2^2 - \mathrm{T}^2\right) = 0.
\end{aligned}
\tag{144}
$$

Energy integral for (144) looks as follows:

$$
\begin{aligned}
&\mathfrak{K} + \mathfrak{U} = \mathrm{E},\\
&\mathfrak{K} = \frac{1}{2}\left((\mathrm{T}^{\bullet})^2 - (\mathrm{U}_2^{\bullet})^2 - (\mathrm{U}_3^{\bullet})^2\right),\\
&\mathfrak{U} = \frac{1}{2}\left(\mathrm{U}_2^2\mathrm{U}_3^2 + \mathrm{T}\left(4\pi - \mathrm{U}_2^2 - \mathrm{U}_3^2\right)\right).
\end{aligned}
$$

Chiral determinant CD $\left(\overset{1}{\boldsymbol{\tau}},\ \overset{2}{\boldsymbol{\tau}},\ \overset{3}{\boldsymbol{\tau}}\right)$ has the form of (104). The "soft chiralization" procedure for orthogonal one-current terriculum wave is similar to the zero-current orthogonal terriculum wave, described above.
We do not provide here a description of a one-current triplet wave with an isotropic wave vector. This description leads to the results which are completely analogous to those given in part 9.2.13 for zero-current triplet wave with isotropic wave vector. Such a wave is a degenerate "algebraic" object which behavior is not controlled by differential equations.

9.3.5 One-neutrino Plane Triplet Wave

The four-dimensional neutrino zone, which state of currents and potentials is described by the one-neutrino Lagrangian (123) and, correspondingly, by the field equations (125) –

(130), must border the zones in which there are anisotropic currents on some unknown in advance three-dimensional boundaries. The conditions for solutions splicing on the three-dimensional boundaries of these different types of 4-zones must allow to determine not only the currents / potentials, but also the Lagrange's multipliers λ. By abstracting from the existence of these boundaries and expanding the area of the one-neutrino Lagrangian (123) over the whole space-time, we get some hypothetical state that does not exist in a "pure form" in nature. Let us name this state a "Yang-Mills neutrino" (*ymino*). In the state of *ymino* the Lagranges multiplier remains arbitrary, obeying only to the condition of orthogonality (132).

Isotropization of any of the four currents of the singlet-triplet theory in some 4-zone generates a neutrino state. Consequently, the ST-theory contains a few types of neutrino: a pure maxwellian neutrino, totally located in the singlet sector; three indistinguishable YM-neutrinos, located in the triplet sector of the theory (*ymino*); and a mixed Maxwell-Yang-Mills neutrino, appearing under the simultaneous isotropization of singlet current J^ν and the third component of YM-current $\overset{3}{\mathrm{J}}{}^{\nu}$. This mixed neutrino state, not fitting into either singlet or triplet sectors, occupies the both sectors of the theory.

It can be assumed that historically determined binding of a neutrino type to a massive lepton type ("electron neutrino", "muonic neutrino", "τ-neutrino") is incorrect since it does not find a natural reflection in the apparatus of the ST-theory. The interpretation of a neutrino as a lepton is also incorrect itself. Within the framework of the theory constructed here, leptons are pure, stationary singlet states. Neutrino states cannot be stationary.

It should be noted that there is no a priori forbidding for the possibility of isotropization of any of the eight currents of the Yang-Mills octuplet which is responsible for the strong interaction. So, there are no reasons to except the existence of "quark" or "gluon" neutrinos, which are not connected with the triplet sector of physics.

We will try a solution to the field equations of one-neutrino state of ymino (125) – (130) in the form of a plane wave with wave vector k^ν. All of the physical variables in a plane wave depend only on wave phase $\phi = k^\nu x_\nu$. The wave equation (125) for the plane wave takes the form:

$$k^\mu k_\mu \mathbf{A}^{\nu\,\prime\prime} + 2k^\mu \mathbf{A}_\mu \times \mathbf{A}^{\nu\,\prime} - k^\nu \mathbf{A}_\mu \times \mathbf{A}^{\mu\,\prime} + \hat{\mathrm{I}}\mathbf{A}^\nu - 4\pi\,\mathrm{p}\,\overset{k}{\mathrm{A}}{}^{\nu}\,\overset{k}{\mathbf{e}} = 0, \tag{145}$$

$$k_\nu \mathbf{A}^{\nu\,\prime} = 0, \tag{146}$$

$$\overset{k}{\mathrm{A}}{}^{\nu}\,\overset{a}{\mathrm{A}}_{\nu} = 0,\ \ (a = 1,\,2,\,3)\,. \tag{147}$$

In these equations $\mathrm{p} = -\dfrac{1}{\lambda}$, $\mathrm{p} = \mathrm{const}$; the prime denotes a wave phase derivative; Yang-Mills inertia tensor $\hat{\mathrm{I}}$ is determined by formula (122); YM-index "k" is "immovable", the summation is not made by it. This index denotes the YM-component which contains the only current of a one-current problem. Further we will fix $k = 1$. If homogeneous equations (145) - (147) are solved, the neutrino current is determined from the condition

$$N_\nu = \mathrm{p}\,\overset{k}{\mathrm{A}}_{\nu}\,.$$

From the condition (146) it follows that

$$k_\nu \mathbf{A}^\nu = \mathbf{q} = \text{const}.$$

As in other wave problems, we will assume that $\mathbf{q} = 0$. If vector $\mathbf{q}$ was nonzero, it would mean that we deal with a neutrino wave on the background of some stationary field. With $\mathbf{q} = 0$ the second term vanishes from the field equation (145):

$$k^\mu k_\mu \mathbf{A}^{\nu\,\prime\prime} - k^\nu \mathbf{A}_\mu \times \mathbf{A}^{\mu\,\prime} + \hat{\mathrm{I}}\mathbf{A}^\nu - 4\pi\, \mathrm{p} \overset{k}{\mathrm{A}}{}^\nu \overset{k}{\mathbf{e}} = 0. \quad (148)$$

By scalar multiplying (148) by wave vector k^ν, with anisotropic wave vector k^ν we find that:

$$\mathbf{A}_\mu \times \mathbf{A}^{\mu\,\prime} = 0. \quad (149)$$

Relation (149) is a standard Yang-Mills nonholonomic constraint, which also appears in the other wave problems studied above. With regard to (149), the equation (148) is simplified:

$$k^\mu k_\mu \mathbf{A}^{\nu\,\prime\prime} + \hat{\mathrm{I}}\mathbf{A}^\nu - 4\pi\, \mathrm{p} \overset{1}{\mathrm{A}}{}^\nu \overset{1}{\mathbf{e}} = 0. \quad (150)$$

From relation (147) follows that potential $\overset{1}{\mathrm{A}}{}^\nu$ is an isotropic 4-vector: $\overset{1}{\mathrm{A}}{}^\nu \overset{1}{\mathrm{A}}_\nu = 0$, or in the notation of the form "1+3":

$$\overset{1}{\mathrm{T}}{}^2 - \overset{1}{\mathbf{U}}{}^2 = 0.$$

Consequently, the three spatial components $\overset{1}{\mathbf{U}}$ of 4-vector $\overset{1}{\mathrm{A}}{}^\nu$ can be formulated as follows:

$$\overset{1}{\mathbf{U}} = \overset{1}{\mathrm{T}}\, \mathbf{e};\ \ \mathbf{e}^2 = 1,$$

where $\mathbf{e}$ is some three-dimensional unit vector. Using wave vector k^ν in the notation of the form "1+3", $k^\nu = \{\omega; \mathbf{k}\}$, the orthogonality condition

$$k^\nu \overset{1}{\mathrm{A}}_\nu = 0$$

can be presented this way:

$$\overset{1}{\mathrm{T}}\, (\omega - \mathbf{k}\mathbf{e}) = 0,$$

or $\omega = \mathbf{k}\mathbf{e}$. But $|\, \mathbf{k}\mathbf{e}\, | \le |\, \mathbf{k}\, |$, so, $\omega^2 \le \mathbf{k}^2$, i.e. the wave vector of the neutrino wave ymino (as well as the wave vector of the maxwellian neutrino wave, considered in the article [1]) cannot be time-like. Leaving aside the degenerate case of isotropic wave vector, let us suppose that

$$k^\nu k_\nu = -k_S^2,$$

where k_S is a Lorentz-invariant pseudo-Euclidean module of space-like wave vector k_ν; $k_S \neq 0$.

By making the scale transformation, which we have also used for the wave problems above,

$$\phi \to k_S \phi,\ \ k^\nu \to k_S k^\nu,$$

we can normalize a wave vector for unit in the field equations (150):

$$-\mathbf{A}^{\nu\,\prime\prime} + \hat{\mathrm{I}}\mathbf{A}^{\nu} - 4\pi\,\mathrm{p}\,\overset{1}{\mathrm{A}}{}^{\nu}\,\overset{1}{\mathbf{e}} = 0, \tag{151}$$

$$k^{\nu}k_{\nu} = -1, \tag{152}$$

$$k^{\nu}\mathbf{A}_{\nu} = 0, \tag{153}$$

$$\overset{1}{\mathrm{A}}{}^{\nu}\mathbf{A}_{\nu} = 0. \tag{154}$$

System (151) – (154) together with nonholonomic constraint (149) makes formulation of the problem of Yang-Mills neutrino – ymino.

By scalar multiplying the field equation (151) by $\mathbf{A}'_{\nu}$ and integrating by the wave phase, we find, with regard to (154), the energy integral of the neutrino ymino wave, which identically coincides with the energy integral of a zero-current wave with a space-like vector. Like for a zero-current wave with nonzero energy, it is possible to rescale the variables per unit energy without changing field equations (151):

$$\mathbf{A}^{\nu} \rightarrow \mid \mathrm{E} \mid^{\frac{1}{4}} \mathbf{A}^{\nu},$$
$$\phi \rightarrow \mid \mathrm{E} \mid^{-\frac{1}{4}} \phi,$$
$$\mathrm{p} \rightarrow \mid \mathrm{E} \mid^{\frac{1}{2}} \mathrm{p}.$$

As for a zero-current wave with space-like wave vector, the energy of a neutrino wave does not have a fixed sign – the energy sign is determined by arbitrarily set initial conditions. It is hardly doubtful that here the reader's relativistic intuition is also ready to protest angrily against the theory in which there appear some objects with negative energy and, therefore, negative mass. But perhaps the appearance of such objects is just a different way of description in the *classical* language of those phenomena which in the language of quantum are described as the production of virtual particles off-mass-shell surface with a virtual violation of the energy conservation law for short time intervals: the shorter this time interval is, the higher, according to the uncertainty relation, the uncertainty in the energy of state is.

For example, the process of transformation of a muon into an electron is described in the language of *quantum* through the transformation of a muon into a *virtual* W-particle with the mass exceeding the muon mass on three decimal orders, with concomitant production of a muon neutrino and fast subsequent transformation of W-particle into an electron and concomitant electron neutrino.

The classical theory, developed here, presupposes the continuous dependence of all physical variables of the theory on the space-time arguments.

It requires a different language to describe such processes, since the classical theory does not allow virtual particles: this is a non-stationary transformation of a *particle* (i.e. an object with the current boundary pomerium) into another, more massive particle, accompanied by generation of a non-stationary neutrino zone with negative energy – in this case the energy conservation law is not violated.

Undoubtedly, there is a considerable distance between this speculative picture and a clear formulation of the initial boundary value problem, describing the muon decay in the classical theory under consideration: it is even unobvious that this distance generally can be surmountable in the *classical* theory framework by combining the total four-current Lagrangian with neutrino Lagrangians in some previously unknown 4-zones. However, the opening possibility to use the language of waves with negative energy an alternative one to the language of virtual particles – seems fruitful in itself.

In the ymino intrinsic frame of reference, in which $k^\nu = \{0;\, 1;\, 0;\, 0\}$, and $\overset{a}{\mathrm{A}}{}^\nu = \{\overset{a}{\mathrm{T}};\, \overset{a}{\mathbf{U}}{}^\nu\}$, from the orthogonality condition (153) follows that all longitudinal potential components vanish:

$$\overset{a}{\mathrm{U}}_1 = 0,$$

and the other 9 components of YM-triplet of potentials are controlled by the system of equations (151), which in notation "1+3" take the following form:

$$v'' + v\left(4\pi \mathrm{p} - \overset{11}{\mathrm{I}}\right) = 0, \tag{155}$$

where $v = \overset{1}{\mathrm{T}},\, \overset{1}{\mathrm{U}}_2,\, \overset{1}{\mathrm{U}}_3$ is any of the three nonzero components of vector $\overset{1}{\mathrm{A}}{}^\nu$, and

$$\overset{11}{\mathrm{I}} = \overset{2}{\mathbf{U}}{}^2 + \overset{3}{\mathbf{U}}{}^2 - \overset{2}{\mathrm{T}}{}^2 - \overset{3}{\mathrm{T}}{}^2,$$

(in vectors $\overset{2}{\mathbf{U}}$ and $\overset{3}{\mathbf{U}}$ the longitudinal components $\overset{2}{\mathrm{U}}_1$ and $\overset{3}{\mathrm{U}}_1$ are missing).

Equations for the two other time components of 4-vector A^μ, according to (151), have the form:

$$\begin{aligned} &\overset{2}{\mathrm{T}}{}'' - \overset{2}{\mathrm{T}}\overset{3}{\mathbf{U}}{}^2 + \overset{3}{\mathrm{T}}\overset{2}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}} = 0, \\ &\overset{3}{\mathrm{T}}{}'' + \overset{2}{\mathrm{T}}\overset{2}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}} - \overset{3}{\mathrm{T}}\overset{2}{\mathbf{U}}{}^2 = 0, \end{aligned} \tag{156}$$

and the equations for spatial components of 4-vector A^μ have the form:

$$\begin{aligned} &\overset{2}{\mathbf{U}}{}'' + \overset{2}{\mathbf{U}}\overset{3}{\mathrm{T}}{}^2 - \overset{3}{\mathbf{U}}\overset{2}{\mathrm{T}}\overset{3}{\mathrm{T}} + \overset{3}{\mathbf{U}} \times \left(\overset{3}{\mathbf{U}} \times \overset{2}{\mathbf{U}}\right) = 0, \\ &\overset{3}{\mathbf{U}}{}'' + \overset{3}{\mathbf{U}}\overset{2}{\mathrm{T}}{}^2 - \overset{2}{\mathbf{U}}\overset{2}{\mathrm{T}}\overset{3}{\mathrm{T}} + \overset{2}{\mathbf{U}} \times \left(\overset{2}{\mathbf{U}} \times \overset{3}{\mathbf{U}}\right) = 0. \end{aligned} \tag{157}$$

The orthogonality conditions (147) in notations "1+3" have the form:

$$\begin{aligned} &\overset{1}{\mathrm{T}}{}^2 - \overset{1}{\mathbf{U}}{}^2 = 0, \\ &\overset{1}{\mathrm{T}}\overset{2}{\mathrm{T}} - \overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}} = 0, \\ &\overset{1}{\mathrm{T}}\overset{3}{\mathrm{T}} - \overset{1}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}} = 0. \end{aligned} \tag{158}$$

Nonholonomic constraints (149) in the language of variables $\overset{a}{\mathrm{T}}$ and $\overset{a}{\mathbf{U}}$ take the form:

$$\overset{2}{\mathrm{T}}\overset{3}{\mathrm{T}}{}' - \overset{3}{\mathrm{T}}\overset{2}{\mathrm{T}}{}' - \left(\overset{2}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}}{}' - \overset{3}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}}{}'\right) = 0. \tag{159}$$

and two more equations are obtained from (159) by means of cyclic permutation of YM-indices: $1 \to 2 \to 3 \to 1$. For a plane-polarized ymino wave, the drawn wave equations allow further simplifications. In such wave all spatial vectors $\overset{a}{\mathbf{U}}$ have fixed directions. By turning axes y and z around axis x, oriented along wave vector $\mathbf{k}$, it is possible to combine axis y with the direction of vector $\overset{1}{\mathbf{U}}$ oscillations.
Then, with regard to (158), we can formulate all the three-dimensional vectors $\overset{a}{\mathbf{U}}$ as follows:

$$\begin{aligned} \overset{1}{\mathbf{U}} &= \{0;\ \overset{1}{\mathrm{T}};\ 0\}, \\ \overset{2}{\mathbf{U}} &= \{0;\ \overset{2}{\mathrm{T}};\ u\}, \\ \overset{3}{\mathbf{U}} &= \{0;\ \overset{3}{\mathrm{T}};\ v\}, \end{aligned} \tag{160}$$

where u and v are z-components of vectors $\overset{1}{\mathbf{U}}$ and $\overset{2}{\mathbf{U}}$. The choice of the axes orientation (160) turns the ymino wave into a "five-component" mathematical object, characterized by five functions $\overset{1}{\mathrm{T}}$, $\overset{2}{\mathrm{T}}$, $\overset{3}{\mathrm{T}}$, u, v.
Subject to the form of vectors $\overset{a}{\mathbf{U}}$ (160), the orthogonality conditions (158) become identities. Field equations (155), (156), (157) subject to (160) are simplified:

$$\overset{1}{\mathrm{T}}{}'' + \overset{1}{\mathrm{T}} \left(4\pi \mathrm{p} - u^2 - v^2\right) = 0, \tag{161}$$

$$\begin{cases} \overset{2}{\mathrm{T}}{}'' - \overset{2}{\mathrm{T}}\, v^2 + \overset{3}{\mathrm{T}}\, uv = 0, \\ \overset{3}{\mathrm{T}}{}'' - \overset{3}{\mathrm{T}}\, v^2 + \overset{2}{\mathrm{T}}\, uv = 0. \end{cases} \tag{162}$$

$$\begin{cases} u'' = 0, \\ v'' = 0. \end{cases} \tag{163}$$

Only trivial solutions to equations (163) are physically acceptable:

$$u = 0;\ \ v = 0. \tag{164}$$

General solution (163) with linear growth of components u and v along longitudinal coordinate x, in accordance with equations (162) and (161), result in exponential growth of time components $\overset{a}{\mathrm{T}}$. For ymino wave we have no possibility to stop the solution growth and to transfer it into an oscillating regime by means of "soft chiralization" procedure, since chiral determinant CD (101) for such a wave is identically zero. So, we have either to accept (164) as the only solution that has a physical meaning, or to assume that the ymino wave turns into a wave with anisotropic current due to some instability which is not reflected in the neutrino Lagrangian (123).
Accepting solution (164), we have to, due to the same reasons, accept a zero solution for components $\overset{2}{\mathrm{T}}$ and $\overset{3}{\mathrm{T}}$, which are controlled by equations (162). Equation (161) for component $\overset{1}{\mathrm{T}}$ turns into the equation of a harmonic oscillator with frequency $\omega = \sqrt{4\pi \mathrm{p}}$ $(\mathrm{p} > 0)$.
What is the physical meaning of the obtained solution?

Subject to (160), the two YM-potential components $\overset{2}{\mathrm{A}}{}^{\nu}$ and $\overset{2}{\mathrm{A}}{}^{\nu}$ vanish, and the component $\overset{2}{\mathrm{A}}{}^{\nu}$ describes the "stiffed" in the wave intrinsic frame of reference sinusoidal ripple, i.e. the ymino wave, nominally located in the triplet sector of physics, is actually a one-component singlet object, which does not differ from the maxwellian neutrino described in the article [1]. When calculating the energy of this one-component ymino by formula (28), applicable to the triplet wave with space-like wave vector, we find that the energy of ymino is equal to zero.

Indeed, ymino wave with such properties is a strange, confusing object. This object is generated by the Lagrangian (123) and we have no a priori reasons to throw ymino away from physics, to recognize it as a non-existent object. Even the total "five component" ymino wave (161), (162) with potentials growing along the wave longitudinal coordinate should probably be considered as a real physical object, as the description of the transition state associated with the decay of one current state with anisotropic current and the production of another current state.

9.3.6 One-current Stationary Triplet State

One-current stationary triplet state is described by the one-current Lagrangian (114) in the zone, occupied by current j^{ν}, and by the zero-current Lagrangian of Yang-Mills free field (50), where the current is missing. At the outer boundary of the current zone (pomerium), the isotropization of current j^{ν} is taking place. At the inner boundary of the current zone (latens) the pseudo-Euclidean module of the space-like current j^{ν} reaches its maximum value (7). In the cavitated tubes of latebra, which have latens as the outer boundary, there is only a free triplet field. At the boundaries of pomerium and latens, 3-current $\mathbf{j}$ has no component normal to the surface. At these boundaries potentials and their space-like coordinate derivatives are continuous.

While deriving the one-current triplet state equations, we will use the Lagrangians (50) and (114), written in Minkowski coordinates. To account the Riemannian space curvature, generated by a high density of energy-momentum in the current zone, the equations of stationary one-current state, presented below, must be rewritten in arbitrary curvilinear coordinates and the metric tensor must be subjugated to the Einstein equations. Accounting of this curvature can significantly change the quantitative characteristics of the solutions, but can hardly influence the fact of the existence or non-existence of a solution.

The equations of one-current stationary state are convenient to write in the intrinsic frame of reference of this state. In this frame, the pomerium and latens boundaries are stationary and the time derivatives of all physical values are equal to zero. Fixation of the frame of reference and separation of time and spatial vector components deprives field equation (53) and (121) of Lorentz and Yang-Mills elegance, but makes them more convenient for numerical investigations.

To write the field equations in the intrinsic frame of reference, it is convenient to introduce into this frame a special mathematical object – Yang-Mills bracket.

Let u and v be two YM-vectors depending on spatial coordinates x_k $(k = 1, 2, 3)$. We do not explicitly write out YM-indices over u and v. Geometrical nature of u and v is inessential: with respect to the transformations of spatial coordinates, u and v may be either scalars or vectors. Let us call Yang-Mills bracket over u and v YM-vector $\{u, v\}$ such that its YM-component of number a is as follows:

$$\overset{a}{\{u,\, v\}} = \overset{abc}{\varepsilon}\, \overset{b}{u}\, \nabla\, \overset{c}{v}\,. \tag{165}$$

In formula (165) ∇ is the gradient operator in spatial coordinates. Yang-Mills bracket, introduced in (165), is a bilinear differential form of the first order.
Using the Yang-Mills bracket, the equations of stationary one-current state in the intrinsic frame of reference, derived from the Lorentz-invariant notation of the field equations (53) and (121), can be written in the following form:
a) In the current zone:

$$\begin{aligned} &\nabla \overset{a}{\mathrm{T}} - 2\overset{a}{\{\mathbf{U},\, \mathrm{T}\}} - \overset{ab}{\mathrm{I}}\,\overset{b}{\mathrm{T}} - 4\pi\,\overset{1}{\mathrm{T}}\,\overset{1a}{\delta} = 0,\\ &\nabla \overset{a}{\mathbf{U}} - 2\overset{a}{\{\mathbf{U},\, \mathbf{U}\}} + \overset{a}{\{\mathrm{U}_k,\, \mathrm{U}_k\}} - \overset{a}{\{\mathrm{T},\, \mathrm{T}\}} - \overset{ab}{\mathrm{I}}\,\overset{b}{\mathbf{U}} - 4\pi\,\overset{1}{\mathbf{U}}\,\overset{1a}{\delta} = 0. \end{aligned} \tag{166}$$

$$\nabla\cdot \overset{a}{\mathbf{U}} = 0. \tag{167}$$

$$\begin{aligned} &\overset{1}{\mathrm{T}}\overset{2}{\mathrm{T}} - \overset{1}{\mathbf{U}}\cdot\overset{2}{\mathbf{U}} = 0,\\ &\overset{1}{\mathrm{T}}\overset{3}{\mathrm{T}} - \overset{1}{\mathbf{U}}\cdot\overset{3}{\mathbf{U}} = 0,\\ &-\left(\frac{1}{2}\mathrm{j}_{\mathrm{T}}\right)^2 \leq \overset{1}{\mathrm{T}}{}^2 - \overset{1}{\mathbf{U}}{}^2 \leq 0. \end{aligned} \tag{168}$$

Yang-Mills inertia tensor has the form

$$\overset{ab}{\mathrm{I}} = \overset{a}{\mathrm{T}}\,\overset{b}{\mathrm{T}} - \overset{a}{\mathbf{U}}\cdot\overset{b}{\mathbf{U}} - \overset{ab}{\delta}\cdot\sum_c \left(\mathrm{T}_c^2 - \mathbf{U}_c^2\right), \tag{169}$$

and, subject to orthogonality conditions (168), it has some zero components inside the current zone:

$$\begin{aligned} &\overset{12}{\mathrm{I}} = \overset{21}{\mathrm{I}} = 0,\\ &\overset{13}{\mathrm{I}} = \overset{31}{\mathrm{I}} = 0. \end{aligned} \tag{170}$$

If equations (166) are solved, the current $\mathrm{j} = \{\rho, \mathbf{j}\}$ is determined by its coupling with potential $\overset{1}{\mathrm{A}}{}^{\nu}$ (116):

$$\begin{aligned} &\rho = -\overset{1}{\mathrm{T}};\\ &\mathbf{j} = -\overset{1}{\mathbf{U}}\,. \end{aligned} \tag{171}$$

b) In current-free zone, potentials $\overset{a}{\mathrm{A}}{}^{\nu}$ obey the equations of the free stationary triplet field, i.e. equations (166) without the last term in each of equations (166), without orthogonality conditions (168) and without the condition of "nulling" of some tensor $\overset{ab}{\mathrm{I}}$ elements (170):

$$\nabla \overset{a}{\mathrm{T}} -2\{\mathbf{U}, \mathrm{T} \overset{a}{\}} - \overset{ab}{\mathrm{I}} \overset{b}{\mathrm{T}}= 0,$$
$$\nabla \overset{a}{\mathbf{U}} -2\{\mathbf{U}, \mathbf{U} \overset{a}{\}} +\{\mathrm{U}_k, \mathrm{U}_k \overset{a}{\}} -\{\mathrm{T}, \mathrm{T} \overset{a}{\}} - \overset{ab}{\mathrm{I}} \overset{b}{\mathbf{U}}= 0. \tag{172}$$

Vectors $\overset{a}{\mathbf{U}}$ obey the condition (167).

) The continuity conditions for $\overset{a}{\mathrm{T}}$, $\overset{a}{\mathbf{U}}$ and their derivatives are fulfilled at the boundaries between zones a and b. The vector component $\overset{1}{\mathbf{U}}$, normal to the boundary, vanishes at these boundaries:

$$\overset{1}{\mathbf{U}} \cdot \mathbf{n} = 0, \tag{173}$$

where $\mathbf{n}$ is the unit vector of normal to the surface.

Vector $\overset{1}{\mathrm{A}}{}^{\nu}$ becomes isotropic on the pomerium:

$$\overset{1}{\mathrm{T}}{}^2 - \overset{1}{\mathbf{U}}{}^2 = 0. \tag{174}$$

The current j^{ν} reaches its peak value j_{T} on latens:

$$\overset{1}{\mathrm{T}}{}^2 - \overset{1}{\mathbf{U}}{}^2 = -\left(\frac{1}{2}\mathrm{j}_{\mathrm{T}}\right)^2 . \tag{175}$$

Pomerium and latens boundaries are not pre-set and must be found in the process of the problem solving.

We would like to remind that the system of twelve field equations (166) or (172) must be written in curvilinear coordinates and supplemented with ten Einstein field equations, as it has been noted previously for the similar stationary problem of electrodynamics [1]. We do not have any version of the theorem of existence / non-existence of the solutions to this problem. Perhaps this is one of the most cumbersome problems in mathematical physics.

It is obvious that the set problem (166) – (175) does not withstand the charge inversion $\overset{a}{\mathrm{T}} \to -\overset{a}{\mathrm{T}}$, $\overset{a}{\mathbf{U}} \to -\overset{a}{\mathbf{U}}$ due to the presence of the Yang-Mills bracket, but it is invariant relative to the charge conjugation method, described in part 4.1, which requires the permutation of YM-indices $1 \leftrightarrow 2$ simultaneously with the charge inversion. This permutation transforms the one-current problem with the current having a YM-number 1 into a similar problem with the current number 2.

The problem of constructing chiral-definite solutions for the stationary one-current problem causes certain difficulties. If we formulate the condition for solution chiral-determinacy in the same form (86), (87) as it was formulated in part 9.2.7 for the wave zero-current problem, i.e. if we require sign-definiteness of the chiral determinant, formed by Yang-Mills triple of spatial components of the 4-potentials, the question will arise: what is to be done when this determinant turns into zero?

Some fixed surfaces in the intrinsic frame of reference of a stationary one-current problem will correspond to zero values of the chiral determinant. At the intersection of such surface we have not only to reverse all the signs of spatial derivatives for all the twelve potential components $\overset{a}{\mathrm{T}}$ and $\overset{a}{\mathbf{U}}$, but also to permute YM-indices $1 \leftrightarrow 2$. Such chiralization procedure looks deformed, but the change of sign for gradients, without YM-indices permutation, does not preserve the form of the equations of a stationary one-current problem.

By agreeing to such chiralization procedure, we distort the very term "one-current state", making it conventional. If in the zone, occupied by the current , there are surfaces on which the chiral CD determinant vanishes, this zone splits into sub-zones occupied by currents $\overset{1}{\mathrm{J}}{}^{\nu}$ or $\overset{2}{\mathrm{J}}{}^{\nu}$. Such picture of one-current stationary state can hardly be called inspiring. However, the choice is limited: we either put up with such picture (and thus, generally speaking, recognize the non-terminality and imperfection of Yang-Mills' description of nature), or we ignore the problem of chiral determinacy and construct continuous solutions to the stationary one-current triplet problem, despite probable violation of the terms of chiral determinacy (87).

The problem of the stationary one-current triplet state (if it has a solution) provides a classical description of a particle[33]. In order not to increase the number of new terms, we shall call this particle a "wark", removing the first sound from the word "quark". In the subsequent article of this series, we shall demonstrate that in the octuplet sector of physics there are one-current states with triplet field shell which contains only three of eight potentials of the octuplet sector[34]. Such stationary states in the octuplet sector of physics are a classical model of quarks. The mathematical formulations of the problems of quark and wark structure completely coincide. The physical difference between the wark and quark particles is that wark is made of currents and potentials of the triplet sector of physics and quark – of currents and potentials of the octuplet sector of physics. The fact of free quarks (and hence, the warks) non-existence, which is known from the experience, means that the corresponding one-current stationary triplet problem has no solution with finite energy. This divergence may be connected with "bad" asymptotic behavior of solutions of the free stationary triplet field equations (172) away from the wark's pomerium, with $r \to \infty$ (r is the distance from the center of the particle wark): components of the potentials do not decrease with increasing, Yang-Mills field tensor components (15) decrease proportionally to r^{-1}, and the integral of YM-field energy density diverges at infinity.

Despite this divergence, the problem of one-current stationary triplet state is of interest. Its solution, which does not allow normalization of energy, can be normalized with respect to a charge and magnetic moment [1].

[33] Or, to be precise, the classical description of some Yang-Mills' particle family, where particles differ from each other in pomerium topology, as well as in a number and topology of cavitated tubes latebrae. This family is a kind of Yang-Mills triplet "parallel" to a singlet lepton family.

[34] While describing such states, a mathematician would say about sub-algebras of Lie algebra, corresponding to the group SU (3).

We cannot say whether there is such a particle in the vast empirical multitude of "elementary" particles which would more or less correspond to the model of wark.

9.4 Two-current Triplet States

9.4.1 Classification and the Lagrangians of Two-current Triplet States

In compliance with the general classification of the singlet-triplet states (p.7 of the present article) and the overall view of the "weak" triplet three-current Lagrangian (49), we can consider the following forms of two-current triplet states: two-current neutrinoless state, two-current one-neutrino state and two-current two-neutrino state.

9.4.1.1 Two-current Neutrinoless State

The third component of YM-current is missing; components $\overset{1}{\mathrm{j}}{}^{\nu}$ and $\overset{2}{\mathrm{j}}{}^{\nu}$ are space-like and orthogonal to each other by (46).
Such state can exist in two forms: in the form of a *wave* that has no boundaries of the current zone, or in the form of a *stationary state* that has common for both currents inner and outer zone boundaries occupied by currents; this stationary state corresponds to a certain particle existing in the triplet sector of physics.
A more general treatment of the two-current problem allows simultaneous existence of two non-overlapping or partly overlapping one-current zones with different pomeriums for current $\overset{1}{\mathrm{j}}{}^{\nu}$ and currents $\overset{2}{\mathrm{j}}{}^{\nu}$: this is the problem of nonstationary interaction of the two particles. This interaction can hardly allow an adequate treatment in the framework of the classical field theory. The Lagrangian of two-current neutrinoless state has the form (see (49)):

$$L_w = -\frac{1}{2}\left(\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{1}{\mathrm{j}}_{\nu} + \overset{2}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}_{\nu}\right) - \left(\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{1}{\mathrm{A}}_{\nu} + \overset{2}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{A}}_{\nu}\right) - \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot\mathbf{A}_{\mu\nu} - \eta\,\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}_{\nu}\,. \tag{176}$$

The last term in (176) is a "penalty" for orthogonality of currents $\overset{1}{\mathrm{j}}{}^{\nu}$ and $\overset{2}{\mathrm{j}}{}^{\nu}$ (46), η is the Lagrange's multiplier.
Probably, the concept of the two-current state per se has scientific sense only in case if the condition of current orthogonality (46) is a "natural" condition for the Lagrangian (176), i.e. in case if the field equations, following from (176), have a solution satisfying (46) at $\eta = 0$. On this basis, while varying (176), we will omit the last term.
The field equations for two-current problem have the form which is similar to the equations of the one-current problem (121) and (116):

$$-\Box\mathbf{A}^{\nu} + 2\mathbf{A}_{\mu}\times\partial^{\mu}\mathbf{A}^{\nu} - \mathbf{A}_{\mu}\times\partial^{\nu}\mathbf{A}^{\mu} + \hat{\mathrm{I}}\mathbf{A}^{\nu} = 4\pi\left(\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{1}{\mathbf{e}} + \overset{2}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathbf{e}}\right); \tag{177}$$

(Yang-Mills equations);

$$\begin{aligned} \overset{1}{\mathrm{j}}{}^{\nu} + \overset{1}{\mathrm{A}}{}^{\nu} &= 0,\\ \overset{2}{\mathrm{j}}{}^{\nu} + \overset{2}{\mathrm{A}}{}^{\nu} &= 0. \end{aligned} \tag{178}$$

(current equations).

Due to current equations, potentials $\overset{1}{\mathrm{A}}{}^{\nu}$ and $\overset{2}{\mathrm{A}}{}^{\nu}$ are space-like and orthogonal to each other.

The current equations (178) are satisfied in the zone occupied by currents. Outside this zone, potentials $\overset{a}{\mathrm{A}}{}^{\nu}$ obey Yang-Mills homogeneous equation (177) with a zero right-hand side.

Differential conditions of the first-order must be added to the field equations (177) and (178): the gauge condition for 4-divergence of potential $\overset{a}{\mathrm{A}}{}^{\nu}$ (117) and the condition for the triplet current (19), from which two orthogonality conditions follow for the two-current problem:

$$\begin{aligned} \overset{1}{\mathrm{A}}{}^{\nu}\ \overset{3}{\mathrm{A}}_{\nu} &= 0, \\ \overset{2}{\mathrm{A}}{}^{\nu}\ \overset{3}{\mathrm{A}}_{\nu} &= 0. \end{aligned} \tag{179}$$

The third orthogonality condition must be added to the conditions (179):

$$\overset{1}{\mathrm{A}}{}^{\nu}\ \overset{2}{\mathrm{A}}_{\nu} = 0, \tag{180}$$

which follows from the condition of current orthogonality (46) and current equations (178).

Under relations (179) and (180) in the zone, occupied by the two currents, all three 4-vectors of YM-potential are orthogonal to each other.

9.4.1.2 Two-current One-neutrino State

In this state, one of YM-triplet currents is identically zero, and one of the other two is isotropic and the second one is space-like[35]. This state can be considered in two variants: $Variant$"1/2"

$$\overset{3}{\mathrm{j}}{}^{\nu} \equiv 0;\ \overset{2}{\mathrm{j}}{}^{\nu} = N^{\nu};\ N^{\nu}N_{\nu} = 0;\ \overset{1}{\mathrm{j}}{}^{\nu} = \mathrm{j}^{\nu};\ \mathrm{j}^{\nu}\mathrm{j}_{\nu} < 0;\ \mathrm{j}^{\nu}N_{\nu} = 0. \tag{181}$$

The last relation in (181) expresses the requirement for currents j^{ν} and $\overset{2}{\mathrm{j}}{}^{\nu}$ orthogonality in pure triplet states (43).

According to (49), the Lagrangian of this state has the form:

$$L_w = -\frac{1}{2}\ \overset{1}{\mathrm{j}}{}^{\nu}\ \overset{1}{\mathrm{j}}_{\nu} - \mathrm{j}^{\nu}\ \overset{1}{\mathrm{A}}_{\nu} - N^{\nu}\ \overset{2}{\mathrm{A}}_{\nu} - \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot\mathbf{A}_{\mu\nu} - \frac{\lambda}{2}N^{\nu}N_{\nu}. \tag{182}$$

The last term in (182) is a "penalty" for neutrino (isotropic) character of current $\overset{2}{\mathrm{j}}{}^{\nu}$, λ is the Lagranges multiplier.

The field equations of this state only slightly differ from the equations of the two-current problem (177) and (178):

$$-\Box\mathbf{A}^{\nu} + 2\mathbf{A}_{\mu}\times\partial^{\mu}\mathbf{A}^{\nu} - \mathbf{A}_{\mu}\times\partial^{\nu}\mathbf{A}^{\mu} + \hat{\mathrm{I}}\mathbf{A}^{\nu} = 4\pi\left(\mathrm{j}^{\nu}\ \overset{1}{\mathbf{e}} + N^{\nu}\ \overset{2}{\mathbf{e}}\right); \tag{183}$$

[35] Current $\overset{3}{\mathrm{J}}{}^{\nu}$ in pure triplet state cannot be a space-like vector.

$$\mathrm{j}^\nu + \overset{1}{\mathrm{A}}{}^\nu = 0,$$
$$\lambda N^\nu + \overset{2}{\mathrm{A}}{}^\nu = 0. \tag{184}$$

Gauge condition (117) must be added to these equations.

Similarly to the neutrinoless two-current problem, the condition of pairwise orthogonality of all the three YM-components of potentials (179), (180) must be added to these equations:

$$\overset{1}{\mathrm{A}}{}^\nu \overset{2}{\mathrm{A}}_\nu = 0,\ \overset{2}{\mathrm{A}}{}^\nu \overset{3}{\mathrm{A}}_\nu = 0,\ \overset{3}{\mathrm{A}}{}^\nu \overset{1}{\mathrm{A}}_\nu = 0. \tag{185}$$

Besides these conditions, the neutrino problem includes isotropy condition $\overset{2}{\mathrm{A}}{}^\nu$:

$$\overset{2}{\mathrm{A}}{}^\nu \overset{2}{\mathrm{A}}_\nu = 0. \tag{186}$$

Relations (185) and (186) specify a set of holonomic constraints for dynamical system (183), (184). These constraints are generated by the conditions of two-current one-neutrino problem and differential conditions for YM-triplet currents (19). The same conditions (19) of the given problem explicitly fix the Lagranges multiplier λ:

$$\lambda = 1. \tag{187}$$

Conditions (185), (186) and (187) appear to be "rigid" and incompatible with condition $\mathrm{j}^\nu \mathrm{j}_\nu < 0$. Therefore, the two-current state does not exist in variant "1/2".

Variant"1/3"

$$\overset{2}{\mathrm{j}}{}^\nu \equiv 0;\ \overset{3}{\mathrm{j}}{}^\nu = N^\nu;\ N^\nu N_\nu = 0;\ \overset{1}{\mathrm{j}}{}^\nu = \mathrm{j}^\nu;\ \mathrm{j}^\nu \mathrm{j}_\nu < 0. \tag{188}$$

In this variant the orthogonality condition of currents j^ν and N^ν vanishes.

The Lagrangian of this state has the form:

$$L_w = -\frac{1}{2}\mathrm{j}^\nu \mathrm{j}_\nu - \mathrm{j}^\nu \overset{1}{\mathrm{A}}_\nu - N^\nu \overset{3}{\mathrm{A}}_\nu - \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot\mathbf{A}_{\mu\nu} - \frac{\lambda}{2}N^\nu N_\nu. \tag{189}$$

The field equations of this problem almost do not differ from the equations of variant "1/2":

$$-\Box\mathbf{A}^\nu + 2\mathbf{A}_\mu \times \partial_\mu \mathbf{A}^\nu - \mathbf{A}_\mu \times \partial^\nu \mathbf{A}^\mu + \hat{\mathrm{I}}\mathbf{A}^\nu = 4\pi\left(\mathrm{j}^\nu \overset{1}{\mathbf{e}} + N^\nu \overset{3}{\mathbf{e}}\right); \tag{190}$$

$$\mathrm{j}^\nu + \overset{1}{\mathrm{A}}{}^\nu = 0,$$
$$\lambda N^\nu + \overset{3}{\mathrm{A}}{}^\nu = 0. \tag{191}$$

Condition for 4-divergence (117) must be added to these equations.

Similarly to variant "1/2", in variant "1/3", according to (19), $\lambda = 1$, and the additional conditions for the potentials analogous to conditions (185) and (186), take the form:

$$\overset{1}{\mathrm{A}}{}^\nu \overset{2}{\mathrm{A}}_\nu = 0,\ \overset{2}{\mathrm{A}}{}^\nu \overset{3}{\mathrm{A}}_\nu = 0,\ \overset{3}{\mathrm{A}}{}^\nu \overset{3}{\mathrm{A}}_\nu = 0. \tag{192}$$

In this variant of the two-current one-neutrino state, the number of holonomic constraints (192), imposed on the solution of the field equations (190) and (191), is one unit less than

in variant "1/2" (185), (186).
It is possible to construct the solution to field equations (190) and (191), satisfying the conditions (192), in the form of a plane wave with a space-like wave vector. This solution grows unrestrictedly along the longitudinal wave coordinate. In this case, the solution growth cannot be stopped through chiralization procedure since the chiral determinant (101) for this wave is identically zero. But the current growth is restricted by the condition (7). Application of this condition allows to construct restricted solutions with derivative discontinuity.

9.4.1.3 Two-current Two-neutrino State

In this state both currents are isotropic. This state can also be considered in two variants: variant "1/2" (isotropic currents $\overset{1}{\mathrm{j}}{}^{\nu}$ and $\overset{2}{\mathrm{j}}{}^{\nu}$, orthogonal to each other, are nonzero), and variant "1/3" (isotropic currents $\overset{1}{\mathrm{j}}{}^{\nu}$ and $\overset{3}{\mathrm{j}}{}^{\nu}$ are nonzero, but their orthogonality is not a necessary condition).
Variant"1/2"

$$\begin{aligned} &\overset{1}{\mathrm{j}}{}^{\nu} = \overset{1}{N}{}^{\nu};\ \overset{2}{\mathrm{j}}{}^{\nu} = \overset{2}{N}{}^{\nu};\ ,\overset{3}{\mathrm{j}}{}^{\nu} \equiv 0. \\ &\overset{1}{N}{}^{\nu}\ \overset{1}{N}_{\nu} = 0;\ \overset{2}{N}{}^{\nu}\ \overset{2}{N}_{\nu} = 0;\ \overset{1}{N}{}^{\nu}\ \overset{2}{N}_{\nu} = 0. \end{aligned} \tag{193}$$

The Lagrangian of this state has the form:

$$L_w = -\ \overset{1}{N}{}^{\nu}\ \overset{1}{\mathrm{A}}_{\nu} -\ \overset{2}{N}{}^{\nu}\ \overset{2}{\mathrm{A}}_{\nu} - \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot\mathbf{A}_{\mu\nu} - \frac{\lambda_1}{2}\ \overset{1}{N}{}^{\nu}\ \overset{1}{N}_{\nu} - \frac{\lambda_2}{2}\ \overset{2}{N}{}^{\nu}\ \overset{2}{N}_{\nu}\ . \tag{194}$$

In (194), λ_1 and λ_2 are the Lagranges multipliers.
The field equations of a two-neutrino problem with the Lagrangian (194) are similar to the field equations of the two-current neutrinoless problem (177) and (178):

$$-\Box\mathbf{A}^{\nu} + 2\mathbf{A}_{\mu}\times\partial^{\mu}\mathbf{A}^{\nu} - \mathbf{A}_{\mu}\times\partial^{\nu}\mathbf{A}^{\mu} + \hat{\mathrm{I}}\mathbf{A}^{\nu} = 4\pi\left(\overset{1}{N}{}^{\nu}\ \overset{1}{\mathbf{e}} + \overset{2}{N}{}^{\nu}\ \overset{2}{\mathbf{e}}\right); \tag{195}$$

$$\begin{aligned} &\lambda_1\ \overset{1}{N}{}^{\nu} + \overset{1}{\mathrm{A}}{}^{\nu} = 0, \\ &\lambda_2\ \overset{2}{N}{}^{\nu} + \overset{2}{\mathrm{A}}{}^{\nu} = 0. \end{aligned} \tag{196}$$

Under the current equations (196) and current conditions (193), potentials $\overset{1}{\mathrm{A}}{}^{\nu}$ and $\overset{2}{\mathrm{A}}{}^{\nu}$ are isotropic and orthogonal to each other:

$$\overset{1}{\mathrm{A}}{}^{\nu}\ \overset{2}{\mathrm{A}}_{\nu} = 0,\ \overset{2}{\mathrm{A}}{}^{\nu}\ \overset{2}{\mathrm{A}}_{\nu} = 0,\ \overset{1}{\mathrm{A}}{}^{\nu}\ \overset{2}{\mathrm{A}}_{\nu} = 0. \tag{197}$$

The differential condition (19), which must be satisfied by YM-triplet currents, will provide three more relations for the physical variables of the two-neutrino problem:

$$\overset{1}{\mathrm{A}}{}^{\nu}\ \overset{3}{\mathrm{A}}_{\nu} = 0,\ \overset{2}{\mathrm{A}}{}^{\nu}\ \overset{3}{\mathrm{A}}_{\nu} = 0,\ \lambda_1 = \lambda_2 = \lambda. \tag{198}$$

The relations (197) and (198) are five holonomic constraints, which are imposed on the 12 components of triplet potential $\overset{a}{\mathrm{A}}{}^{\nu}$, satisfying Yang-Mills equations (195). The Lagranges multiplier λ remains arbitrary in this problem.

It is easy to see that the wave with at least one isotropic current can have only a space-like wave vector. In the intrinsic system of such wave, we can try solutions to the field equations (195) as follows:

$$\overset{1}{\mathrm{A}}{}^{\nu} = \left\{ \overset{1}{\mathrm{T}},\, 0,\, \overset{1}{\mathrm{T}},\, 0 \right\},$$
$$\overset{2}{\mathrm{A}}{}^{\nu} = \left\{ \overset{2}{\mathrm{T}},\, 0,\, \overset{2}{\mathrm{T}},\, 0 \right\}, \tag{199}$$
$$\overset{3}{\mathrm{A}}{}^{\nu} = \left\{ \overset{3}{\mathrm{T}},\, 0,\, \overset{3}{\mathrm{T}},\, v \right\},$$

Potentials of the form (199) satisfy the holonomic constraints (197), (198) and the non-holonomic constraints which appear in the plane wave theory. The substitution of (199) into (195) gives, subject to (196) and (198):

$$\begin{aligned} &\overset{1}{\mathrm{T}}{}''+ \overset{1}{\mathrm{T}} \left(4\pi \mathrm{p} - v^2\right) = 0,\\ &\overset{2}{\mathrm{T}}{}''+ \overset{2}{\mathrm{T}} \left(4\pi \mathrm{p} - v^2\right) = 0. \end{aligned} \tag{200}$$

$$\begin{aligned} &\overset{3}{\mathrm{T}}{}'' = 0,\\ &v'' = 0, \end{aligned} \tag{201}$$

where $\mathrm{p} = -\dfrac{1}{\lambda}$, and the prime stands for the longitudinal coordinate x derivative.

The general solution (201) gives linear growth of components $\overset{3}{\mathrm{T}}$ and v along the longitudinal coordinate. With substitution of this solution into (200), we get the exponential growth of components $\overset{1}{\mathrm{T}}$ and $\overset{2}{\mathrm{T}}$. This growth cannot be stopped by means of chiralization procedure, since the chiral determinant CD (101) is identically zero for this wave.

So, we have to choose only a trivial solution (201): $\overset{3}{\mathrm{T}}= 0$, $v = 0$. In this case, potential $\overset{3}{\mathrm{A}}{}^{\nu}$ vanishes, and the two-component couple $\overset{1}{\mathrm{A}}{}^{\nu}$ and $\overset{2}{\mathrm{A}}{}^{\nu}$, in accordance with (200), at $\mathrm{p} > 0$ splits into two independent harmonious waves – two non-interacting singlet maxwellian neutrinos with the same spatial period.

$Variant$"1/3"

$$\begin{aligned} &\overset{1}{\mathrm{j}}{}^{\nu} = \overset{1}{N}{}^{\nu};\ \overset{2}{\mathrm{j}}{}^{\nu} \equiv 0;\ ,\overset{3}{\mathrm{j}}{}^{\nu} = \overset{3}{N}{}^{\nu}.\\ &\overset{1}{N}{}^{\nu}\ \overset{1}{N}_{\nu}= 0;\ \overset{3}{N}{}^{\nu}\ \overset{3}{N}_{\nu}= 0. \end{aligned} \tag{202}$$

The Lagrangian of this state has the following form:

$$L_w = -\overset{1}{N}{}^{\nu}\ \overset{1}{\mathrm{A}}_{\nu} - \overset{3}{N}{}^{\nu}\ \overset{3}{\mathrm{A}}_{\nu} - \frac{1}{16\pi}\mathbf{A}^{\mu\nu}\cdot\mathbf{A}_{\mu\nu} - \frac{\lambda_1}{2}\ \overset{1}{N}{}^{\nu}\ \overset{1}{N}_{\nu} - \frac{\lambda_3}{2}\ \overset{3}{N}{}^{\nu}\ \overset{3}{N}_{\nu}\ . \tag{203}$$

Field equations, generated by the Lagrangian (203), get the following form:

$$-\Box\mathbf{A}^{\nu} + 2\mathbf{A}_{\mu}\times\partial^{\mu}\mathbf{A}^{\nu} - \mathbf{A}_{\mu}\times\partial^{\nu}\mathbf{A}^{\mu} + \hat{\mathrm{I}}\mathbf{A}^{\nu} = 4\pi\left(\overset{1}{N}{}^{\nu}\ \overset{1}{\mathbf{e}} + \overset{3}{N}{}^{\nu}\ \overset{3}{\mathbf{e}}\right); \tag{204}$$

$$\begin{aligned} \lambda_1 \overset{1}{N}{}^{\nu} + \overset{1}{\mathrm{A}}{}^{\nu} = 0, \\ \lambda_3 \overset{3}{N}{}^{\nu} + \overset{3}{\mathrm{A}}{}^{\nu} = 0. \end{aligned} \tag{205}$$

Potentials $\overset{1}{\mathrm{A}}{}^{\nu}$ and $\overset{2}{\mathrm{A}}{}^{\nu}$ are isotropic:

$$\overset{1}{\mathrm{A}}{}^{\nu} \overset{1}{\mathrm{A}}_{\nu} = 0, \; \overset{3}{\mathrm{A}}{}^{\nu} \overset{3}{\mathrm{A}}_{\nu} = 0. \tag{206}$$

Differential relations for the triplet currents (19) generate three other conditions for the state (202), (205):

$$\overset{1}{\mathrm{A}}{}^{\nu} \overset{2}{\mathrm{A}}_{\nu} = 0, \; \overset{2}{\mathrm{A}}{}^{\nu} \overset{3}{\mathrm{A}}_{\nu} = 0, \; \lambda_1 = \lambda_3 = \lambda. \tag{207}$$

The Lagranges multiplier λ remains arbitrary in this state.
Trying partial solution (204), (205) in the form of a plane wave with a wave vector , we can suggest that

$$\begin{aligned} \overset{1}{\mathrm{A}}{}^{\nu} &= \left\{ \overset{1}{\mathrm{T}}, 0, \overset{1}{\mathrm{T}}, 0 \right\}, \\ \overset{2}{\mathrm{A}}{}^{\nu} &= \left\{ \overset{2}{\mathrm{T}}, 0, \overset{2}{\mathrm{T}}, v \right\}, \\ \overset{3}{\mathrm{A}}{}^{\nu} &= \left\{ \overset{3}{\mathrm{T}}, 0, \overset{3}{\mathrm{T}}, 0 \right\}, \end{aligned} \tag{208}$$

When choosing a potential in the form (208), the holonomic constraints (206), (207), as well as nonholonomic ones, will be satisfied, and the field equations (204) with (205) give the system of equations which is completely analogous to the system (200), (201) $\mathrm{p} = -\dfrac{1}{\lambda}$:

$$\begin{aligned} \overset{1}{\mathrm{T}}{}'' + \overset{1}{\mathrm{T}} \left(4\pi\mathrm{p} - v^2\right) = 0, \\ \overset{3}{\mathrm{T}}{}'' + \overset{3}{\mathrm{T}} \left(4\pi\mathrm{p} - v^2\right) = 0. \end{aligned} \tag{209}$$

$$\begin{aligned} \overset{2}{\mathrm{T}}{}'' = 0, \\ v'' = 0. \end{aligned} \tag{210}$$

For the same reasons that have been formed for variant "1/2", we have to choose the trivial solution (210):
$\overset{3}{\mathrm{T}} = 0$, $v = 0$, at which potential $\overset{2}{\mathrm{A}}{}^{\nu}$ vanishes, and potentials $\overset{1}{\mathrm{A}}{}^{\nu}$ and $\overset{3}{\mathrm{A}}{}^{\nu}$ form a couple of interacting singlet harmonic waves (a couple of maxwellian neutrino) with spatial frequency $\sqrt{4\pi\mathrm{p}}$.

9.4.2 Two-current Triplet Neutrinoless State as a Plane Wave

9.4.2.1 Triplet Plane Wave with a Time-like Wave Vector and Linear Polarization

Let us write the equations of two-current neutrinoless wave in the intrinsic frame of reference of the wave, supposing that the time-like wave vector is normalized for a unit.

In this frame of reference all the time components of the potentials and currents are vanishing, and the spatial components of potentials $\overset{a}{\mathbf{U}}$, subject to (179) and (180) are orthogonal to each other. Considering the wave with linear polarization, we can direct the spatial coordinate axes in their intrinsic wave system so that each of the three vectors $\overset{a}{\mathbf{U}}$ would oscillate along the same coordinate axis. It allows us to use one Cartesian index for numbering all the potential components while omitting YM-indices:

$$\overset{a}{\mathbf{U}}_i = 0 \text{ for } i \neq a; \;\; \overset{a}{\mathbf{U}}_i \equiv u_i \text{ for } i = a. \tag{211}$$

For the three values of u_i introduced by the relation (211) from Yang-Mills equations (177) and current equations (178), we can obtain the following system of differential equations:

$$\begin{aligned} &\ddot{u}_1 + \left(4\pi + u_2^2 + u_3^2\right) u_1 = 0, \\ &\ddot{u}_2 + \left(4\pi + u_3^2 + u_1^2\right) u_2 = 0, \\ &\ddot{u}_3 + \left(u_1^2 + u_2^2\right) u_3 = 0. \end{aligned} \tag{212}$$

The dots in the equations (212) denote differentiation with respect to the intrinsic time of the wave. While choosing potential components in the form (211), the holonomic constraints (179) and (180) are satisfied. The nonholonomic constraints, imposed on the triplet potential, are also satisfied. For the two-current wave, these constraints have the same form as for the zero-current wave. The chiral determinant CD for the wave (211) has the form

$$\mathrm{CD} = u_1\, u_2\, u_3,$$

and, correspondingly, the condition of chiral determinacy for the system (212) has the form:

$$u_1\, u_2\, u_3 \geq 0. \tag{213}$$

The procedure of "soft" chiralization for this wave is the replacement of of those instants of time when at least one of the values vanishes. The procedure of "hard" chiralization would be a permutation of the Cartesian indices $1 \leftrightarrow 2$ in solutions to the system of equations (212) at these instants of time. The system (212) "does not notice" such permutation. The dynamical system (212), which we call "two-current Yang-Mills oscillator", has an obvious energy integral:

$$\frac{1}{2}\left(\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2\right) + \frac{1}{2}\left(u_1^2\, u_2^2 + u_2^2\, u_3^2 + u_3^2\, u_1^2 + 4\pi\left(u_1^2 + u_2^2\right)\right) = \mathrm{E}. \tag{214}$$

System (212) is not self-similar by energy. We cannot normalize the wave energy (214) per unit by scale transformation. Therefore, for a two-current wave, unlike a zero-current wave, it is possible to talk about the "wave of high energy" or the "wave of low energy". The wave of high energy behaves like a zero-current wave. For the wave of low energy in the potential wave energy, the values of the fourth order can be neglected in contrast to the values of the second order. Yang-Mills two-current oscillator (if we ignore the condition of chiral determinacy and not provide a "soft" chiralization) generally has a chaotic behavior. However, there are also initial conditions, under which periodic solutions to the equations (212) appear. "Soft" chiralization "kills" the chaos, turning chaotic solutions into periodic, but does it at the expense of appearance of derivative discontinuities $\dot{u}_i$.

9.4.2.2 Doublet Plane Wave with Time-like Wave Vector and Circular Polarization

Along with linear-polarized wave (212) it is possible to consider degenerate doublet wave in which $\overset{3}{\mathrm{A}}{}^{\nu} \equiv 0$, and vectors $\overset{1}{\mathbf{U}}$ and $\overset{2}{\mathbf{U}}$ rotate around axis being orthogonal to each other and to the axis:

$$\begin{aligned} &\overset{1}{\mathbf{U}} = \mathrm{X}\left\{0,\, \cos\psi,\, \sin\psi\right\};\\ &\overset{2}{\mathbf{U}} = \mathrm{X}\left\{0,\, -\sin\psi,\, \cos\psi\right\};\\ &\overset{3}{\mathbf{U}} = 0. \end{aligned} \tag{215}$$

In relations (215) X is the wave amplitude, ψ is the phase angle of the wave. The amplitude and phase angle depend on time.

Yang-Mills equations (177) and (178) for the doublet wave (215) take the following form:

$$\ddot{\mathrm{X}} + \mathrm{X}\left(4\pi + \mathrm{X}^2 - \dot{\psi}^2\right) = 0, \tag{216}$$

$$\mathrm{X}^2\dot{\psi} = \mathrm{M} = \mathrm{const}. \tag{217}$$

The integral of the doublet wave M (217) can be identified with the angular momentum of this wave. Substitution of (217) into the equation of motion (216) transforms the problem of the doublet wave into quite a simple one-dimensional problem:

$$\ddot{\mathrm{X}} + \mathrm{X}\left(4\pi + \mathrm{X}^2 - \frac{\mathrm{M}^2}{\mathrm{X}^4}\right) = 0, \tag{218}$$

One-dimensional oscillator (218) has an obvious energy integral:

$$\begin{aligned} &\mathfrak{K} + \mathfrak{U} = \mathrm{E} = \mathrm{const},\\ &\mathfrak{K} = \frac{1}{2}\dot{\mathrm{X}}^2,\\ &\mathfrak{U} = 2\pi\mathrm{X}^2 + \frac{\mathrm{X}^4}{4} + \frac{\mathrm{M}^2}{2\mathrm{X}^2}. \end{aligned} \tag{219}$$

The problem (218) describes periodic radial oscillations of doublet vectors in the potential well with potential energy $\mathfrak{U}$ (219), followed by rotation of vectors around axis x according to (217). The energy of the doublet wave cannot be less than the minimum value of $\mathrm{E}_{\min}$:

$$\mathrm{E}_{\min} = 2\pi\mathrm{X}_*^2 + \frac{\mathrm{X}_*^4}{4} + \frac{\mathrm{M}^2}{2\mathrm{X}_*^2},$$

where X_*, in accordance with (218), is the root of the equation

$$\mathrm{X}_*^6 + 4\pi\mathrm{X}_*^4 = \mathrm{M}^2.$$

In the state with minimum energy, doublet vectors $\overset{1}{\mathbf{U}}$ and $\overset{2}{\mathbf{U}}$, rotate around axis with a constant angular velocity $\dot{\psi}$, conserving a constant length.
This doublet problem is simply appealing for the naive Bohrs wave angular momentum quantization. But, probably, it would be premature to leave the territory of Yang-Mills classical waves, not fully opened by us yet, for the sake of this "Bohr adventure".

9.4.2.3 Two-current Triplet Neutrinoless State as a Plane Wave with a Space-like Wave Vector and Linear Polarization

For a two-current neutrinoless wave with space-like wave vector $k^\nu = \{0;\, 1;\, 0;\, 0\}$, normalized for unit, Yang-Mills field equations (177), subject to the current equations (178), can be written as follows:

$$\begin{aligned} \overset{1}{\boldsymbol{\tau}}{}'' - \overset{1}{\boldsymbol{\tau}} \left(4\pi - \overset{2}{\boldsymbol{\tau}}{}^2 - \overset{3}{\boldsymbol{\tau}}{}^2\right) &= 0, \\ \overset{2}{\boldsymbol{\tau}}{}'' - \overset{2}{\boldsymbol{\tau}} \left(4\pi - \overset{3}{\boldsymbol{\tau}}{}^2 - \overset{1}{\boldsymbol{\tau}}{}^2\right) &= 0, \\ \overset{3}{\boldsymbol{\tau}}{}'' - \overset{3}{\boldsymbol{\tau}} \left(- \overset{1}{\boldsymbol{\tau}}{}^2 - \overset{2}{\boldsymbol{\tau}}{}^2\right) &= 0, \end{aligned} \tag{220}$$

where $\overset{a}{\boldsymbol{\tau}} = \left\{\overset{a}{\mathrm{T}};\, i\, \overset{a}{\mathbf{U}}\right\}$ are the three-dimensional complex YM-vectors, that have been introduced here before, in which spatial components $\overset{a}{\mathbf{U}}$ are orthogonal to the wave direction:

$$\overset{a}{\mathbf{U}} = \left\{0;\, \overset{a}{\mathrm{U}}_2;\, \overset{a}{\mathrm{U}}_3\right\}.$$

The primes in (220) stand for differentiation along the longitudinal coordinate .
Non-holonomic constraints for the waves (220) have the following form:

$$\overset{abc}{\varepsilon}\, \overset{b}{\boldsymbol{\tau}} \cdot \overset{c}{\boldsymbol{\tau}}{}' = 0,$$

and holonomic constraints (179) and (180) take the form:

$$\overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}} = 0,\ \overset{2}{\boldsymbol{\tau}} \cdot \overset{3}{\boldsymbol{\tau}} = 0,\ \overset{3}{\boldsymbol{\tau}} \cdot \overset{1}{\boldsymbol{\tau}} = 0,$$

or:

$$\begin{aligned} \overset{1}{\mathrm{T}} \cdot \overset{2}{\mathrm{T}} - \overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}} &= 0, \\ \overset{2}{\mathrm{T}} \cdot \overset{3}{\mathrm{T}} - \overset{2}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}} &= 0, \\ \overset{3}{\mathrm{T}} \cdot \overset{1}{\mathrm{T}} - \overset{3}{\mathbf{U}} \cdot \overset{1}{\mathbf{U}} &= 0. \end{aligned}$$

Obviously, these equations are compatible only in case of satisfying the condition of solvability

$$D \geq 0,$$

where $D = \left(\overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}}\right) \left(\overset{2}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}}\right) \left(\overset{3}{\mathbf{U}} \cdot \overset{1}{\mathbf{U}}\right)$,

or $\cos\alpha_{12} \cdot \cos\alpha_{23} \cdot \cos\alpha_{31} \geq 0$,

where α_{ab} is the angle between vectors $\overset{a}{\mathbf{U}}$ and $\overset{b}{\mathbf{U}}$.
If $D \neq 0$, the equations of holonomic constraints can be solved relative to time components of potentials $\overset{a}{\mathrm{T}}$:

$$\overset{1}{\mathrm{T}} = \pm\frac{\sqrt{D}}{\left|\overset{2}{\mathrm{U}} \cdot \overset{3}{\mathrm{U}}\right|};\ \overset{2}{\mathrm{T}} = \pm\frac{\sqrt{D}}{\left|\overset{3}{\mathrm{U}} \cdot \overset{1}{\mathrm{U}}\right|};\ \overset{3}{\mathrm{T}} = \pm\frac{\sqrt{D}}{\left|\overset{1}{\mathrm{U}} \cdot \overset{2}{\mathrm{U}}\right|}.$$

In these relations, the sign of one of values $\overset{a}{\mathrm{T}}$ can be chosen arbitrarily, the signs of the other two time components are determined by the equations of holonomic constraints.
These explicitly written relations for $\overset{a}{\mathrm{T}}$ can be used instead of differential equations for $\overset{a}{\mathrm{T}}$ of the system of nine equations of motion (220), using only six spatial components of vectors $\overset{a}{\boldsymbol{\tau}}$ at the numerical solution of (220).
Nonholonomic constraints must be taken into account only at the formation of the initial conditions for the system (220) with some arbitrarily chosen value of longitudinal coordinate x.
The condition for chiral determinacy of such wave is formed as a condition, imposed on complex vectors $\overset{a}{\boldsymbol{\tau}}$ (see formulas (100) and (101)).
It is easy to construct a partial solution to this problem, formed by three orthogonal vectors $\overset{a}{\boldsymbol{\tau}}$, in which vectors $\overset{a}{\boldsymbol{\tau}}$ have the following form:

$$\begin{aligned}\overset{1}{\boldsymbol{\tau}} &= \{0;\ 0;\ iv\},\\ \overset{2}{\boldsymbol{\tau}} &= \{0;\ iu;\ 0\},\\ \overset{3}{\boldsymbol{\tau}} &= \{\mathrm{T};\ 0;\ 0\}.\end{aligned}$$

For this type of wave, the equations of holonomic and nonholonomic constraints are identically satisfied, and the dynamic equations (220) take the following form:

$$\begin{aligned}&\mathrm{T}'' - \mathrm{T}\ \left(u^2 + v^2\right) = 0,\\ &v'' - v\ \left(4\pi + u^2 - \mathrm{T}^2\right) = 0,\\ &u'' - u\ \left(4\pi + v^2 - \mathrm{T}^2\right) = 0.\end{aligned} \tag{221}$$

Energy integral for system (220) has the form:

$$\frac{1}{2}\left((\mathrm{T}')^2 - (u')^2 - (v')^2\right) + \frac{1}{2}\left(u^2v^2 + \left(4\pi - \mathrm{T}^2\right)\ \left(u^2 + v^2\right)\right) = \mathrm{E}.$$

The energy of such wave is not positively determined.
Chiral determinant for such orthogonal wave has the form:

$$\mathrm{CD} = \mathrm{T}\, u\, v.$$

The procedure of soft chiralization of the solutions to system (221) consists in a sign change of derivatives T', u', v', with those values of the longitudinal coordinate , for which at least one of functions T, u or v, vanishes.
The same procedure must be applied when one of the two or both currents reach their limiting value, i.e. the signs of derivatives T', u', v' are changed if $|u| = \dfrac{1}{2}\,\mathrm{j_T}$ or $|v| = \dfrac{1}{2}\,\mathrm{j_T}$.

9.4.2.4 Doublet Plane Wave with Space-like Wave Vector and Circular Polarization

A doublet wave with circular polarization can be considered as a partial solution to the equations of motion (220). In this wave, the third YM-component of potential $\overset{3}{\mathrm{A}}{}^{\nu}$ is identically zero (i.e. $\overset{3}{\boldsymbol{\tau}}\equiv 0$), and doublet $\overset{1}{\boldsymbol{\tau}}$, $\overset{2}{\boldsymbol{\tau}}$ can be written in a form similar to (215):

$$\overset{1}{\boldsymbol{\tau}}= i\mathrm{X}\{0;\ \cos\psi;\ \sin\psi\},$$
$$\overset{2}{\boldsymbol{\tau}}= i\mathrm{X}\{0;\ -\sin\psi;\ \cos\psi\},$$

where $\mathrm{X} = \mathrm{X}\,(x)$ is the doublet wave amplitude and $\psi = \psi\,(x)$ is the phase angle of the doublet wave. With phase angle change, vectors $\overset{1}{\boldsymbol{\tau}}$ and $\overset{2}{\boldsymbol{\tau}}$ rotate around the wave direction, while remaining orthogonal to axis and to each other. Yang-Mills equations (220) for the doublet wave look as follows:

$$\mathrm{X}'' - \mathrm{X}\left(4\pi + \mathrm{X}^2 + (\psi')^2\right) = 0; \tag{222}$$

$$\mathrm{X}^2\psi' = \mathrm{M} = \mathrm{const.} \tag{223}$$

Constant M in the integral of motion (223) can be identified with the angular momentum of circularly – polarized wave in the intrinsic system of the wave.
By extracting angular velocity ψ' from (222) with help of (223), we reduce the problem of the doublet wave to some one-dimensional amplitude problem:

$$\mathrm{X}'' - \mathrm{X}\left(4\pi + \mathrm{X}^2 + \frac{\mathrm{M}^2}{\mathrm{X}^4}\right) = 0; \tag{224}$$

Equation (224) has an obvious energy integral:

$$\frac{1}{2}\,(\mathrm{X}')^2 - 2\pi\mathrm{X}^2 - \frac{\mathrm{X}^4}{4} + \frac{\mathrm{M}^2}{2\mathrm{X}^2} = -\mathrm{E} = \mathrm{const.} \tag{225}$$

Equations (224) and (225) describe a wave with amplitude X, infinitely growing along the longitudinal wave coordinate. However, the growth of the current is limited by the condition (7):

$$|\mathrm{X}| \le \frac{1}{2}\mathrm{j_T}. \tag{226}$$

When in relation (226) the sign of equality is reached at a certain value of the longitudinal coordinate $x = x_*$, the current growth ceases, and we have no right to continue solving the equation (224) uninterruptedly further along the longitudinal coordinate of the wave with $x > x_*$.
Here we find ourselves faced with a choice of one of two options, A and B:
A) We can assume that, with $x > x_*$, currents $\overset{1}{\mathrm{j}}$ and $\overset{2}{\mathrm{j}}$ are missing, and there is only a free zero-current doublet stationary Yang-Mills field which is available;
B) We can accept that with $x = x_*$ there is a sign change of derivatives $\overset{1}{\boldsymbol{\tau}}{}'$ and $\overset{2}{\boldsymbol{\tau}}{}'$, i.e. $\mathrm{X}' \to -\mathrm{X}'$ and $\psi' \to -\psi'$. Wave energy does not change, but the angular momentum

changes its sign.
By accepting the alternative A, we assume that an external observer can see the surface $x = x_*$, on which the current takes its maximum value.
We tend to introduce some exclusion principle, which requires that the surface of the current maximum value should be only within the current zone, and would be inaccessible for an external observer located outside the current zone. Accepting this exclusion principle, we have to reject the alternative A and to accept the alternative B, which allows discontinuity of the derivatives of currents and potentials, and even discontinuity (non-conservation) of the angular momentum of the wave. This is a hard and unpleasant choice: since Maxwell's time the classical field theory has always dealt only with continuous solutions.
The alternative B turns a continuous and unlimitedly growing solution of the equation (224), into periodic ripple: the amplitude of doublet wave X oscillates along the wave direction, from some certain minimum value, reached at $\mathrm{X}' = 0$, up to the maximum value (226). Wave energy, according to (225) can be either positive or negative. Equations (224) and (225), together with condition B, give, probably, qualitatively correct description of the doublet wave with circular polarization. For correct quantitative description, it is necessary to take into account the space curvature generated by the wave itself with large amplitude of current X.

9.4.3 Stationary Two-current State

For stationary two-current state, there is an allocated intrinsic frame of reference in which neither current nor field components depend on time. The position and the shape of current zone boundaries (the outer boundary – pomerium and internal boundaries – latens) do not depend on time as well. We will assume that these boundaries are common for the two orthogonal currents of the two-current state.
The equations, describing distribution of fields in the current zone, can be borrowed from the equations of a one-current stationary problem, by adding the terms, generated by the presence of the second current, to equations (166):

$$\begin{aligned} &\Delta \overset{a}{\mathrm{T}} -2\{\mathbf{U},\, \mathrm{T} \overset{a}{\}} - \overset{ab}{\mathrm{I}}\overset{b}{\mathrm{T}} -4\pi \left(\overset{1}{\mathrm{T}}\overset{1a}{\delta} + \overset{2}{\mathrm{T}}\overset{2a}{\delta} \right) = 0, \\ &\Delta \overset{a}{\mathbf{U}} -2\{\mathbf{U},\, \mathbf{U} \overset{a}{\}} +\{\mathrm{U}_k,\, \mathrm{U}_k \overset{a}{\}} -\{\mathrm{T},\, \mathrm{T} \overset{a}{\}} - \overset{ab}{\mathrm{I}}\overset{b}{\mathbf{U}} -4\pi \left(\overset{1}{\mathbf{U}}\overset{1a}{\delta} + \overset{2}{\mathbf{U}}\overset{2a}{\delta} \right) = 0. \end{aligned} \tag{227}$$

In these equations, as before, $\overset{a}{\mathrm{T}}$ is the time components of YM-potentials in the intrinsic frame of reference, $\overset{a}{\mathbf{U}}$ is their spatial components; $\{u,\, v \overset{a}{\}}$ is Yang-Mills bracket (165), introduced before, $\overset{ab}{\mathrm{I}}$ is ,Yang-Mills inertia tensor (169).
Similarly to the one-current problem, the conditions for vector $(\mathbf{j})$ (167) divergence must be added to Yang-Mills stationary equations (227):

$$\nabla \cdot \overset{a}{\mathbf{U}} = 0.$$

Instead of the two orthogonality conditions (168) that were present in the one-current problem, in the two-current problem there are three of such conditions (179), (180) imposing three holonomic constraint equations to the twelve components of the triplet of potentials $\overset{a}{\mathrm{A}}{}^{\nu}$:

$$\begin{aligned} \overset{1}{\mathrm{T}}\overset{2}{\mathrm{T}} - \overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}} &= 0, \\ \overset{2}{\mathrm{T}}\overset{3}{\mathrm{T}} - \overset{2}{\mathbf{U}} \cdot \overset{3}{\mathbf{U}} &= 0, \\ \overset{3}{\mathrm{T}}\overset{1}{\mathrm{T}} - \overset{3}{\mathbf{U}} \cdot \overset{1}{\mathbf{U}} &= 0. \end{aligned} \tag{228}$$

These orthogonality conditions mean that Yang-Mills inertia tensor $\overset{ab}{\mathrm{I}}$ is diagonal:

$$\overset{ab}{\mathrm{I}} = 0 \quad a \neq b.$$

Currents $\overset{1}{\mathrm{j}}{}^{\nu}$ and $\overset{2}{\mathrm{j}}{}^{\nu}$, and, consequently, the corresponding YM-components of potentials, are space-like and module-limited everywhere in the current zone:

$$\begin{aligned} -\left(\frac{1}{2}\mathrm{j}_{\mathrm{T}}\right)^2 \leq \overset{1}{\mathrm{T}}{}^2 - \overset{1}{\mathbf{U}}{}^2 \leq 0, \\ -\left(\frac{1}{2}\mathrm{j}_{\mathrm{T}}\right)^2 \leq \overset{2}{\mathrm{T}}{}^2 - \overset{2}{\mathbf{U}}{}^2 \leq 0, \end{aligned} \tag{229}$$

and the limiting values for the first and second currents are reached simultaneously, at the boundaries of the current zone which are common for two currents.

We tend to believe that this joint achievement of the limiting current values will be provided only if on currents (and potentials) there is imposed one more holonomic constraint, which would require the equality of pseudo-Euclidean modules of the two currents, not only at the boundaries, but everywhere in the current zone:

$$\overset{1}{\mathrm{T}}{}^2 - \overset{1}{\mathbf{U}}{}^2 = \overset{2}{\mathrm{T}}{}^2 - \overset{2}{\mathbf{U}}{}^2. \tag{230}$$

This condition[36] means that Yang-Mills inertia tensor $\overset{ab}{\mathrm{I}}$ has axial symmetry:

$$\overset{11}{\mathrm{I}} = \overset{22}{\mathrm{I}} .$$

Similarly to the one-current problem, if the equations (227) are solved and the components of YM-triplet potentials are found, time (ρ) and spatial ($\mathbf{j}$) components of YM-doublet of currents of the two-current problem can be determined by the current equations (110):

$$\begin{aligned} \overset{1}{\rho} = -\overset{1}{\mathrm{T}}; \ \overset{1}{\mathbf{j}} = -\overset{1}{\mathbf{U}}; \\ \overset{2}{\rho} = -\overset{2}{\mathrm{T}}; \ \overset{2}{\mathbf{j}} = -\overset{2}{\mathbf{U}} . \end{aligned} \tag{231}$$

[36] We have no existence theorem for a two-current stationary problem available, and we cannot conclude whether the system of four holonomic constraints (228) and (230) is too tough and blocks the existence of solutions.

In the zone, free of the currents, (i.e. inside the cavitated tubes latebrae, restricted by the current boundary latens, and outside the outer boundary of the current zone- pomerium), potentials $\overset{a}{\mathrm{A}}{}^{\nu}$ obey the equations of a free stationary triplet field (172), without holonomic constraints (228) and (230), and without restrictions on pseudo-Euclidean module (229). At the boundaries of the current zone, the spatial components of currents do not have a component which would be normal to the boundary:

$$\overset{1}{\mathbf{U}} \cdot \mathbf{n} = 0;\ \overset{2}{\mathbf{U}} \cdot \mathbf{n} = 0, \tag{232}$$

where $\mathbf{n}$ is a unit normal vector to the surface.
At the pomerium boundary, both of the currents are isotropic:

$$\overset{1}{\mathrm{T}}{}^{2} - \overset{1}{\mathbf{U}}{}^{2} = \overset{2}{\mathrm{T}}{}^{2} - \overset{2}{\mathbf{U}}{}^{2} = 0,$$

and at the latens boundary, both of the currents rech their maximum value(175):

$$\overset{1}{\mathrm{T}}{}^{2} - \overset{1}{\mathbf{U}}{}^{2} = \overset{2}{\mathrm{T}}{}^{2} - \overset{2}{\mathbf{U}}{}^{2} = -\left(\frac{1}{2}\mathrm{j}_{\mathrm{T}}\right)^{2}.$$

For two-current stationary state we can literally reproduce everything that has been said above for the one-current state due to the need for consideration of the Riemannian space curvature (not reflected in the equations (227)) and due to the need for consideration of the chiral determinacy condition (87).
The considered formulation of the problem of the two-current stationary state (if this problem has a solution) provides a classical description of a certain particle[37]. We do not know whether some real particle corresponds to this hypothetical model. In the octuplet sector of physics there can be also set a problem of the stationary two-current state in the triplet shell of potentials. This problem is described by the same system of partial differential equations (227) with the same holonomic constraints (228), (230)), as the two-current problem in the triplet sector of physics, considered above. Probably, some real particles – charged pions and other charged "two-quark" bosons – correspond to stationary two-current states of the octuplet sector of physics. We can not specify any known particles, resembling pions of the octuplet sector, in the triplet sector of physics[38].

9.5 Three-current Triplet States

9.5.1 One-neutrino Three-current Triplet State

In three-current state, one of the currents – $\overset{3}{\mathrm{j}}{}^{\nu}$ – is neutrino. Two other currents are space-like and orthogonal to each other. However, with variation of the "weak Lagrangian" (49), we will omit term $\eta\, \overset{1}{\mathrm{j}}{}^{\nu}\, \overset{2}{\mathrm{j}}{}^{\nu}$ – "the penalty for currents orthogonality". We will adhere to the minimalist position: either it is possible to construct solutions to the field equations

[37] Or, to be precise, a particle family. See the similar application for a one-current problem.
[38] Except for W-bosons, but they are enormously massive in comparison to pions.

that arise with variation of the Lagrangian (49) with zero Lagranges multiplier η, or the corresponding three-current states cannot exist in a "pure" form, being transitional between the states with other current formulas.

The states, containing neutrino current, cannot be stationary. However, the nonstationarity of these states can manifest itself in different ways. For example, a set of three-current triplet states includes the state with distinct current zones for currents $\overset{1}{\mathrm{j}}{}^{\nu}$ and $\overset{2}{\mathrm{j}}{}^{\nu}$: there is a triplet one-current zone with current $\overset{1}{\mathrm{j}}{}^{\nu}$ and another triplet current zone with current $\overset{2}{\mathrm{j}}{}^{\nu}$; each of these zones has its outer boundary – pomerium, and the Lagrangian (49) describes the interaction of these two zones (two "triplet particles") through the third neutrino current.

Description of these "exchange" interactions within the framework of the classical field theory may turn out to be incomparable to any particular experimental situation. But the classical description of all triplet wave states that do not have a pomerium, seems to be relevant – as relevant as the description of the singlet wave states, which include ordinary electromagnetic wave and also a "heavy photon" and "maxwellian neutrino", considered in the article above [1]. Considering three-current triplet state as a plane wave, we can use the weak Lagrangian (49) and Yang-Mills equations which arise from this Lagrangian:

$$-\Box\mathbf{A}^{\nu} + 2\mathbf{A}_{\mu}\times\partial^{\mu}\mathbf{A}^{\nu} - \mathbf{A}_{\mu}\times\partial^{\nu}\mathbf{A}^{\mu} + \hat{\mathrm{I}}\mathbf{A}_{\nu} = 4\pi\mathbf{J}^{\nu}. \tag{233}$$

In equation (233), the triplet of YM-currents, according to (49), has the form:

$$\begin{gathered}\mathbf{J}^{\nu} = \left\{\overset{1}{\mathrm{j}}{}^{\nu};\, \overset{2}{\mathrm{j}}{}^{\nu};\, N^{\nu}\right\};\\ \overset{1}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}{}^{\nu} = 0;\;\; N^{\nu}N_{\nu} = 0.\end{gathered} \tag{234}$$

Current equations, supplementing Yang-Mills systems of equations (233), have the following form for the three-current problem:

$$\begin{gathered}\overset{1}{\mathrm{j}}{}^{\nu} + \overset{1}{\mathrm{A}}{}^{\nu} = 0;\\ \overset{2}{\mathrm{j}}{}^{\nu} + \overset{2}{\mathrm{A}}{}^{\nu} = 0;\\ \lambda N^{\nu} + \overset{3}{\mathrm{A}}{}^{\nu} = 0.\end{gathered} \tag{235}$$

From relations (234) and (235) follows that potentials $\overset{a}{\mathrm{A}}{}^{\nu}$ obey the following constraint relations:

$$\overset{1}{\mathrm{A}}{}^{\nu}\;\overset{2}{\mathrm{A}}_{\nu} = 0;\;\; \overset{3}{\mathrm{A}}{}^{\nu}\;\overset{3}{\mathrm{A}}_{\nu} = 0. \tag{236}$$

The potentials also obey the gauge condition by 4-divergence:

$$\partial_{\nu}\,\overset{a}{\mathrm{A}}{}^{\nu} = 0. \tag{237}$$

Besides this, physical variables of the three-current problem obey the differential relation (19):

$$\partial_\nu \mathbf{J}^\nu + \mathbf{A}_\nu \times \mathbf{J}^\nu = 0. \tag{238}$$

Comparison of (238) with (237), (236) and (235) allows to fix the undetermined Lagranges multiplier :

$$\lambda = 1. \tag{239}$$

Since neutrino current N^ν is present in the formation of the three-current triplet state, the corresponding plane wave can only have a space-like wave vector k^ν $(k^\nu k_\nu < 0)$. This vector can be normalized to unit by scale transformation. In the wave intrinsic frame of reference, vector k^ν has the form:

$$k^\nu = \{0;\ 1;\ 0;\ 0\}\,. \tag{240}$$

Subject to (237) and (240), none of YM-potentials $\overset{a}{\mathrm{A}}{}^\nu$ has a longitudinal component:

$$\overset{a}{\mathrm{A}}{}^1 = 0. \tag{241}$$

Considering (241), Yang-Mills equations (233) for a plane wave with wave vector (240) are easy to write in the form of the equation for Yang-Mills triple of three-dimensional complex vectors $\overset{a}{\boldsymbol{\tau}}= \left\{\overset{a}{\mathrm{T}};\ i\ \overset{a}{\mathbf{U}}\right\}$, introduced above, ($\overset{a}{\mathrm{T}}$ is a time component of potential $\overset{a}{\mathrm{A}}{}^0$, and $\overset{a}{\mathbf{U}}$ is two-dimensional vectors of spatial components of potential $\overset{a}{\mathrm{A}}$, orthogonal to the wave direction):

$$\begin{aligned}
&\overset{1}{\boldsymbol{\tau}}{}'' - \left(4\pi - \overset{2}{\boldsymbol{\tau}}{}^2\right)\overset{1}{\boldsymbol{\tau}} - \left(\overset{1}{\boldsymbol{\tau}}\cdot\overset{3}{\boldsymbol{\tau}}\right)\overset{3}{\boldsymbol{\tau}} = 0,\\
&\overset{2}{\boldsymbol{\tau}}{}'' - \left(4\pi - \overset{1}{\boldsymbol{\tau}}{}^2\right)\overset{2}{\boldsymbol{\tau}} - \left(\overset{2}{\boldsymbol{\tau}}\cdot\overset{3}{\boldsymbol{\tau}}\right)\overset{3}{\boldsymbol{\tau}} = 0,\\
&\overset{3}{\boldsymbol{\tau}}{}'' - \left(4\pi - \overset{1}{\boldsymbol{\tau}}{}^2 - \overset{2}{\boldsymbol{\tau}}{}^2\right)\overset{3}{\boldsymbol{\tau}} - \left(\overset{1}{\boldsymbol{\tau}}\cdot\overset{3}{\boldsymbol{\tau}}\right)\overset{1}{\boldsymbol{\tau}} - \left(\overset{2}{\boldsymbol{\tau}}\cdot\overset{3}{\boldsymbol{\tau}}\right)\overset{2}{\boldsymbol{\tau}} = 0.
\end{aligned} \tag{242}$$

Holonomic constraints for vectors $\overset{a}{\boldsymbol{\tau}}$ (234) have the following form:

$$\overset{3}{\boldsymbol{\tau}}{}^2 = 0,\ \overset{1}{\boldsymbol{\tau}}\cdot\overset{2}{\boldsymbol{\tau}} = 0. \tag{243}$$

Nonholonomic constraints, resulted from Yang-Mills equations (233), have the form:

$$\overset{abc}{\varepsilon}\,\overset{b}{\boldsymbol{\tau}}\cdot\overset{c}{\boldsymbol{\tau}}{}' = 0, \tag{244}$$

It is sufficient to account the constraints (244) while forming the initial conditions for the system of motion (242) at some initial value of longitudinal coordinate .
Energy integral for the system (242) has the form:

$$+\frac{1}{2}\sum_a\left(\overset{a}{\boldsymbol{\tau}}{}'\right)^2 - \frac{1}{2}\left(4\pi\left(\overset{1}{\boldsymbol{\tau}}{}^2 + \overset{2}{\boldsymbol{\tau}}{}^2\right) + \left(\overset{1}{\boldsymbol{\tau}}\cdot\overset{3}{\boldsymbol{\tau}}\right)^2 + \left(\overset{2}{\boldsymbol{\tau}}\cdot\overset{3}{\boldsymbol{\tau}}\right)^2 - \overset{1}{\boldsymbol{\tau}}{}^2\cdot\overset{2}{\boldsymbol{\tau}}{}^2\right) = \mathrm{E}. \tag{245}$$

Energy E in (245) can have any sign.
While constructing solutions for the systems (242), we also have to take into account the restrictions imposed on the currents (5) and (7):

$$\begin{aligned} -\left(\frac{1}{2}\mathrm{j_T}\right)^2 &\leq \overset{1}{\boldsymbol{\tau}}{}^2 < 0, \\ -\left(\frac{1}{2}\mathrm{j_T}\right)^2 &\leq \overset{1}{\boldsymbol{\tau}}{}^2 < 0, \end{aligned} \tag{246}$$

It is also necessary to take into account the condition for chiral determinacy, imposed on chiral determinant CD, composed of three YM-vectors $\overset{a}{\boldsymbol{\tau}}$ (101).
Conditions (246) and (101) can be treated as unilateral ("releasing") holonomic constraints, imposed on the solution of the system (242). As it is known, such unilateral constraints generate solutions which have derivatives discontinuities.
For problem (242) it is not difficult to construct a degenerate doublet solution, supposing that $\overset{3}{\boldsymbol{\tau}} \equiv 0$. This solution has been considered above (p. 9.4.2.4). We have not managed to construct any other, vector $\overset{3}{\boldsymbol{\tau}}$ nondegenerate, partial solutions to the problem (242), compatible with conditions (243) and inequalities (246). However, we also have not managed to prove formally the statement of nonexistence of the three-current solutions to the equations (242), which obey conditions (243) and inequalities (246).

9.5.2 Three-neutrino Three-current Triplet State

Let us consider such a triplet state, where all of the three currents are neutrino:

$$\begin{aligned} &\overset{1}{\mathrm{j}}{}^{\nu} = \overset{1}{N}{}^{\nu};\ \overset{2}{\mathrm{j}}{}^{\nu} = \overset{2}{N}{}^{\nu};\ \overset{3}{\mathrm{j}}{}^{\nu} = \overset{3}{N}{}^{\nu}; \\ &\overset{1}{N}{}^{\nu}\ \overset{1}{N}_{\nu} = 0;\ \overset{2}{N}{}^{\nu}\ \overset{2}{N}_{\nu} = 0;\ \overset{3}{N}{}^{\nu}\ \overset{3}{N}_{\nu} = 0;\ \overset{1}{N}{}^{\nu}\ \overset{2}{N}_{\nu} = 0. \end{aligned} \tag{247}$$

Upon obtaining Yang-Mills equations for this condition in the basic Lagrangian (49), we have to omit the "current part" of L_w and to add a "penalty" to the basic weak Lagrangian (49) for isotropy of all the three currents of L_{ad}:

$$L_{ad} = -\frac{1}{2}\lambda_1\ \overset{1}{N}{}^{\nu}\ \overset{1}{N}_{\nu} - \frac{1}{2}\lambda_2\ \overset{2}{N}{}^{\nu}\ \overset{2}{N}_{\nu} - \frac{1}{2}\lambda_3\ \overset{3}{N}{}^{\nu}\ \overset{3}{N}_{\nu}, \tag{248}$$

where λ_1, λ_2, λ_3 are the Lagranges multipliers.
The term L_{ad} (248) restores the current part in the Lagrangian, but with arbitrary multipliers λ_a.
Yang-Mills field equations and the current equations can be obtained from the weak Lagrangian (49) subject to L_{ad} (248). Yang-Mills equations in the neutrino problem coincide with the equations (233), at the notation of the right sides (i.e. current) on the basis of (247). The current equation with regard to (248) have the form:

$$\begin{aligned} &\lambda_1\ \overset{1}{N}{}^{\nu} + \overset{1}{\mathrm{A}}_{\nu} = 0; \\ &\lambda_2\ \overset{2}{N}{}^{\nu} + \overset{2}{\mathrm{A}}_{\nu} = 0; \\ &\lambda_3\ \overset{3}{N}{}^{\nu} + \overset{3}{\mathrm{A}}_{\nu} = 0. \end{aligned} \tag{249}$$

From the current equations (249) and differential current condition (237), the statement of the equality of all Lagranges multipliers λ_a can be obtained as a consequence:

$$\lambda_1 = \lambda_2 = \lambda_3 \ (= \lambda)\,. \tag{250}$$

For a plane three-neutrino wave with wave vector (240), the differential equations for Yang-Mills triple of three-dimensional complex vectors $\overset{a}{\boldsymbol{\tau}}$ can be obtained from the field equations (233) and current equations (249):

$$\begin{aligned} &\overset{1}{\boldsymbol{\tau}}{}'' + 4\pi\,\mathrm{p}\ \overset{1}{\boldsymbol{\tau}} - \left(\overset{1}{\boldsymbol{\tau}} \cdot \overset{3}{\boldsymbol{\tau}}\right) \overset{3}{\boldsymbol{\tau}} = 0,\\ &\overset{2}{\boldsymbol{\tau}}{}'' + 4\pi\,\mathrm{p}\ \overset{2}{\boldsymbol{\tau}} - \left(\overset{2}{\boldsymbol{\tau}} \cdot \overset{3}{\boldsymbol{\tau}}\right) \overset{3}{\boldsymbol{\tau}} = 0,\\ &\overset{3}{\boldsymbol{\tau}}{}'' + 4\pi\,\mathrm{p}\ \overset{3}{\boldsymbol{\tau}} - \left(\overset{1}{\boldsymbol{\tau}} \cdot \overset{3}{\boldsymbol{\tau}}\right) \overset{1}{\boldsymbol{\tau}} - \left(\overset{2}{\boldsymbol{\tau}} \cdot \overset{3}{\boldsymbol{\tau}}\right) \overset{2}{\boldsymbol{\tau}} = 0. \end{aligned} \tag{251}$$

In these equations $\mathrm{p} = -1/\lambda$ (λ remains undefined). Nonholonomic constraints (243) are imposed on the solution of the system (251).

These equations (251) are structurally identical to the equations (242), but they account the neutrino character of the currents:

$$\overset{1}{\boldsymbol{\tau}}{}^2 = 0;\ \overset{2}{\boldsymbol{\tau}}{}^2 = 0;\ \overset{3}{\boldsymbol{\tau}}{}^2 = 0. \tag{252}$$

and the orthogonality condition of $\overset{1}{\boldsymbol{\tau}}$ and $\overset{2}{\boldsymbol{\tau}}$:

$$\overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}} = 0. \tag{253}$$

It is easy to construct a partial "three-fold degenerate" solution for the system (251), where all three YM-potentials coincide with each other[39].

$$\overset{1}{\boldsymbol{\tau}} = \overset{2}{\boldsymbol{\tau}} = \overset{3}{\boldsymbol{\tau}}\ . \tag{254}$$

In this solution, all three complex isotropic vectors are at the same time parallel (254) and orthogonal (252) to each other - the geometry is quite strange for those who are used to real vectors in the Euclidean space. Each of the vectors obeys the elementary oscillator equation:

$$\overset{a}{\boldsymbol{\tau}}{}'' + 4\pi\mathrm{p}\ \overset{a}{\boldsymbol{\tau}} = 0, \tag{255}$$

which, with $\mathrm{p} > 0$, describes a sinusoidal ripple with spatial frequency $\sqrt{4\pi\mathrm{p}}$, that is stationary in the intrinsic frame of the wave.

The solution (254), (255) describes a system of three non-interacting and not noticing each other maxwellian neutrinos, described in the article [1]. The Lagrangian parameter p remains undefined in this solution: it is controlled by the processes which occur outside the neutrino zone[40].

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[39] In spite of the fact that in this solution $\overset{1}{\mathrm{A}}{}^{\nu} = \overset{2}{\mathrm{A}}{}^{\nu} = \overset{3}{\mathrm{A}}{}^{\nu}$, we have to suggest that each of the three YM-potentials $\overset{a}{\mathrm{A}}{}^{\nu}$ and each of the three YM-currents $\overset{a}{N}{}^{\nu}$ can be measured independently from each other by means of different (and still unknown) measuring devices: if you do not believe it, there is no need for you to study the theory of Yang-Mills fields.

[40] We did not manage to construct any solutions to the system (251), which are less trivial than (254), (255).

10. Composite Wave Singlet-triplet States

10.1 One-current Composite Singlet-Triplet Neutrino State (Maxwell-Yang-Mills Neutrino – mymino)

10.1.1 The mymino Lagrangian and Field Equations

The simplest singlet-triplet state contains only one isotropic current N^ν which is simultaneously located in the two sectors of physics. This current is the singlet current and the third component of YM-triplet of currents:

$$\mathrm{J}^\nu = N^\nu;\ \overset{3}{\mathrm{J}}{}^\nu = -N^\nu;\ N^\nu N_\nu = 0. \tag{256}$$

Two other components of YM-triplet of currents in the neutrino state do not exist. We will call the state (256) "Maxwell-Yang-Mills neutrino" or *mymino*. Using the basic Lagrangian of the singlet-triplet theory (17) and taking into account (256), the Lagrangian *mymino* can be presented as follows:

$$L_{mym} = -\frac{1}{2\,p_S\,p_T}\,N_\nu Z^\nu - \frac{1}{16\pi}\left(\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} + \mathbf{W}_{\mu\nu}\cdot\mathbf{W}_{\mu\nu}\right) - \frac{\lambda}{4\,p_S\,p_T}N^\nu N_\nu. \tag{257}$$

In the Lagrangian (257), potential Z^ν is a linear combination of singlet potential W^ν and the third component of YM-potential $\overset{3}{\mathrm{W}}{}^\nu$:

$$Z^\nu = p_T\,\mathrm{W}^\nu - p_S\ \overset{3}{\mathrm{W}}{}^\nu. \tag{258}$$

Potential Z^ν appears in (257), since current N^ν interacts with both singlet potential W^ν and the third component of YM- triplet of potentials.
Field tensors $\mathrm{W}^{\mu\nu}$ and $\mathbf{W}^{\mu\nu}$ are determined by the relations (14) and (15).
The last term in (257) was introduced as a "penalty" for the isotropy of current N^ν. In this term λ is the undefined Lagranges factor. The denominator $4p_Sp_T$ is introduced into this term for the relative easiness of further calculations.
Occurrence of the last term in (257) with the Lagranges multiplier λ, which is necessary from the mathematical point of view for the problem of conditional extremum of the action functional, causes certain concern in terms of physical interpretation of the theory. It is equivalent to the occurrence of some external field ("Higgs-like-field"), affecting current directly so as to ensure that it is isotropic. The external field itself is not affected by physical fields and currents occurring in the problem.
Probably, it should be assumed that the neutrino state mymino, described by the Lagrangian (257), can not occupy the whole four-dimensional space – time, but can exist only in a finite and non-stationary four-dimensional area which is a buffer between the areas filled with space-like currents J^ν and $\mathbf{J}^\nu$ and the areas of free fields W^ν and $\mathbf{W}^\nu$ which contain no currents J^ν and $\mathbf{J}^\nu$. Such "multi-zone" problem would describe, in classical language, the process of neutrino exchange between some of the massive particles. The formulation of such "multi-zone" problem is not considered in the article. We treat

the mymino state as a model problem, extending the scope of the Lagrangian (257) application all over the space-time continuum. In such model problem, the "Lagrange's field" λ becomes an uninterpreted external parameter, a kind of a "constraint force". Varying the action functional with the Lagrangian (257) by current N^ν and potentials W^ν, we obtain the following equations that describe the mymino state:

$$\lambda N^\nu + Z^\nu = 0 \quad (\text{"current equation"}), \tag{259}$$

$$\partial_\mu \mathrm{W}^{\mu\nu} = \frac{2\pi}{p_S} N^\nu \quad (\text{"maxwellian equations"}), \tag{260}$$

$$\partial_\mu \mathbf{W}^{\mu\nu} + \mathbf{W}_\mu \times \mathbf{W}^{\mu\nu} = -\frac{2\pi}{p_T} N^\nu \overset{3}{\mathbf{e}} \quad (\text{"Yang-Mills equations"}). \tag{261}$$

In equation (261) $\overset{3}{\mathbf{e}}$ is the unit vector of the third YM- direction with YM-components $\{0;\ 0;\ 1\}$.
Mymino problem solution must satisfy, besides the recorded field equations, the following conditions:

$$\partial_\nu N^\nu = 0 \quad (\text{current conservation}), \tag{262}$$

$$\partial_\nu \mathrm{W}^\nu = 0,\ \partial_\nu \mathbf{W}^\nu = 0 \quad (\text{zero 4-divergence gauge of potentials}). \tag{263}$$

Besides these differential equations, the algebraic orthogonality condition, resulting from missing of currents $\overset{1}{\mathrm{J}}{}^\nu$, $\overset{2}{\mathrm{J}}{}^\nu$ and the differential condition (19) for Yang-Mills triplet of currents, are imposed on the solution of the mymino problem:

$$N^\nu \overset{a}{\mathrm{W}}_\nu = 0,\ (a = 1,\ 2)\,,$$

or, subject to (259):

$$Z^\nu \overset{a}{\mathrm{W}}_\nu = 0,\ (a = 1,\ 2)\,. \tag{264}$$

By means of current equation (259) and the definition of compound Z-field (258), current N^ν and maxwellian singlet W^ν can be excluded from consideration, reducing the problem of mymino to the solution of the system of nonlinear wave equations for YM-triplet $\mathbf{W}^\nu$ and singlet-triplet compound potential Z^ν:

$$\begin{aligned} &-\Box Z^\nu + \frac{2\pi \mathrm{q}}{p_S p_T} Z^\nu = -p_T p_S \left(\overset{3}{\mathrm{h}}{}^\nu + p_T \overset{3b}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu \right); \\ &-\Box \overset{a}{\mathrm{W}}{}^\nu = p_T \left(\overset{a}{\mathrm{h}}{}^\nu + p_T \overset{ab}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu + \frac{2\pi \mathrm{q}}{p_T^2} Z^\nu \overset{3a}{\delta} \right);\ (a = 1,\ 2,\ 3)\,. \end{aligned} \tag{265}$$

where $\mathrm{q} = -\dfrac{1}{\lambda}$;

$$\mathbf{h}^\nu = \mathbf{W}_\mu \times (2\partial^\mu \mathbf{W}^\nu - \partial^\nu \mathbf{W}^\mu)\,, \tag{266}$$

$\mathbf{h}^\nu$ is an auxiliary object (YM-vector and Lorentz vector), vanishing for plane waves.
All additional algebraic and differential conditions of the problem, in terms of four potentials Z^ν, $\mathbf{W}^\nu$, entering into (265), take the following form:

$$\begin{aligned} &\partial_\nu Z^\nu = 0, && \text{(a)}\\ &\partial_\nu \mathbf{W}^\nu = 0, && \text{(b)}\\ &Z^\nu Z_\nu = 0, && \text{(c)}\\ &Z^\nu \overset{a}{\mathrm{W}}_\nu \quad (a=1,\ a=2)\,. && \text{(d)} \end{aligned} \tag{267}$$

10.1.2 Mymino as a Plane Wave

As for the other wave problems, considered above, we shall try the solution for a mymino problem in the form of a plane wave. For such solution, all the problem variables depend only on a single scalar argument ϕ ("wave phase"):

$$\phi = k^\mu x_\mu,$$

where k^μ is a wave vector.
Wave equations (265) for the plane wave take the following form:

$$\begin{aligned} &k_S^2 \left(Z^\nu\right)^{\bullet\bullet} + \frac{2\pi\mathrm{q}}{p_S p_T} Z^\nu = -p_T p_S \left(\overset{3}{\mathrm{h}}{}^\nu + p_T \overset{3b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu\right);\\ &k_S^2 \left(\overset{a}{\mathrm{W}}{}^\nu\right)^{\bullet\bullet} = p_T \left(\overset{a}{\mathrm{h}}{}^\nu + p_T \overset{ab}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu + \frac{2\pi\mathrm{q}}{p_T^2} Z^\nu \overset{3a}{\delta}\right);\ (a=1,\,2,\,3)\,. \end{aligned} \tag{268}$$

$$\begin{aligned} k_\nu \left(Z^\nu\right)^\bullet = 0,\\ k_\nu \left(\mathbf{W}^\nu\right)^\bullet = 0. \end{aligned} \tag{269}$$

In these equations the "dot" symbol, following the variable, means a wave phase ϕ differentiation; k_S^2 is a Lorentz invariant of wave vector k_ν:

$$k^\nu k_\nu = -k_S^2. \tag{270}$$

Wave vector k_ν in the mymino wave, containing isotropic vectors N^ν and Z^ν, can be only space-like.
As in the other wave problems, we shall omit the differentiation symbol in the equations (269), assuming that there are no external constant fields:

$$\begin{aligned} k_\nu Z^\nu = 0,\\ k_\nu \mathbf{W}^\nu = 0. \end{aligned} \tag{271}$$

Vectors $\mathbf{h}^\nu$ for the plane wave, according to the definition of $\mathbf{h}^\nu$ (266), have the form:

$$\mathbf{h}^\nu = 2k^\mu \mathbf{W}_\mu \times \left(\mathbf{W}^\nu\right)^\bullet - k^\nu \mathbf{W}_\mu \times \left(\mathbf{W}^\mu\right)^\bullet. \tag{272}$$

In view of (271), the first term in (272) vanishes. The convolution of the equations (268) with wave vector demonstrates, subject to (271), that the second term in (272) should also vanish:

$$\mathbf{W}_\mu \times \left(\mathbf{W}^\mu\right)^\bullet = 0. \tag{273}$$

As in other wave problems, the equations (273) are nonholonomic constraints, imposed on the initial conditions of the problem.
Vanishing of vector $\mathbf{h}^\nu$ simplifies the mymino equation (268):

$$\begin{aligned} k_S^2 (Z^\nu)^{\bullet\bullet} + \frac{2\pi \mathrm{q}}{p_S p_T} Z^\nu &= -p_S p_T^2 \overset{3b}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu; \\ k_S^2 \left(\overset{a}{\mathrm{W}}{}^\nu\right)^{\bullet\bullet} &= p_T^2 \overset{ab}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu + \frac{2\pi \mathrm{q}}{p_T} Z^\nu \overset{3a}{\delta}. \end{aligned} \tag{274}$$

While solving wave equations (274), it is convenient to use the mymino intrinsic frame of reference, in which wave vector has the only non-zero component directed along axis :

$$k^\nu = k_S \{0;\ 1;\ 0;\ 0\}. \tag{275}$$

In this frame $\phi = k_S x$ and in field equations (274) it is convenient to move from phase ϕ differentiation to the differentiation of longitudinal coordinate :

$$(Z^\nu)^\bullet = \frac{1}{k_S} (Z^\nu)';\ \left(\overset{a}{\mathrm{W}}{}^\nu\right)^\bullet = \frac{1}{k_S} \left(\overset{a}{\mathrm{W}}{}^\nu\right)',$$

where the prime stands for coordinate differentiation. The wave equations take the form:

$$\begin{aligned} (Z^\nu)'' + \frac{2\pi \mathrm{q}}{p_S p_T} Z^\nu &= -p_S p_T^2 \overset{3b}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu; \\ \left(\overset{a}{\mathrm{W}}{}^\nu\right)'' &= p_T^2 \overset{ab}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu + \frac{2\pi \mathrm{q}}{p_T} \overset{3a}{\delta} Z^\nu. \end{aligned} \tag{276}$$

From conditions (271) follows that in the mymino intrinsic frame of reference, the longitudinal components of potentials disappear:

$$Z^1 = 0;\ \overset{a}{\mathrm{W}}{}^1 = 0.$$

The simplest partial solution of a mymino problem can be constructed by assuming that potentials $\overset{1}{\mathrm{W}}{}^\nu$ and $\overset{2}{\mathrm{W}}{}^\nu$ are missing, and potentials Z^ν and $\overset{3}{\mathrm{W}}{}^\nu$ are isotropic. Trying the solution for the problem in the form of linear-polarized wave:

$$\begin{aligned} Z^\nu &= \{\mathrm{T};\ 0;\ \mathrm{T};\ 0\}; \\ \overset{3}{\mathrm{W}}{}^\nu &= \left\{\overset{3}{\mathrm{T}};\ 0;\ \overset{3}{\mathrm{T}};\ 0\right\}, \end{aligned} \tag{277}$$

we find that all holonomic and nonholonomic constraints for the solution of the form (277) are satisfied, Yang-Mills inertia tensor is identically zero, and the dynamic system (276) turns into a couple of linear differential equations:

$$\mathrm{T}'' + \frac{2\pi \mathrm{q}}{p_S p_T} \mathrm{T} = 0; \tag{278}$$

$$\overset{3}{\mathrm{T}}{}'' = \frac{2\pi \mathrm{q}}{p_T} \mathrm{T}. \tag{279}$$

From (278) follows that, with q > 0:

$$\mathrm{T} = \mathrm{Q} \cdot \sin\left(x\sqrt{\frac{2\pi \mathrm{q}}{p_S p_T}} + \alpha \right), \tag{280}$$

where Q and α are arbitrary constants.
Since the solution of a mymino problem should depend only on phase $k_S x$, from (210) can be concluded that the pseudo-Euclidean module of the wave number of a mymino wave has a specified value:

$$k_S^2 = \frac{2\pi \mathrm{q}}{p_S p_T}. \tag{281}$$

But due to the Lagranges multiplier q indeterminacy, the relation (281) can hardly be subjected to any experimental trial.
By choosing for $\overset{3}{\mathrm{T}}$ the solution to the equation (279), bounded at infinity, we can present the partial solution of the mymino problem, constructed here, in the following form:

$$\begin{aligned} &\overset{1}{\mathrm{W}}{}^{\nu} = 0; \quad \overset{2}{\mathrm{W}}{}^{\nu} = 0; \\ &Z^{\nu} = \mathrm{Q}\xi^{\nu}\mathrm{f}; \quad N^{\nu} = \mathrm{Qq}\xi^{\nu}\mathrm{f}; \\ &\mathrm{W}^{\nu} = \mathrm{Q}p_T\xi^{\nu}\mathrm{f}; \ \overset{3}{\mathrm{W}}{}^{\nu} = -\mathrm{Q}p_S\xi^{\nu}\mathrm{f}. \end{aligned} \tag{282}$$

In formulas (282) f-phase wave function is $\mathrm{f} = \sin(k_S x + \alpha)$; $\xi^{\nu} = \{1;\ 0;\ 1;\ 0\}$ is isotropic 4-vector, which specifies 4-orientation of the linear-polarized mymino wave, Q is the arbitrary amplitude multiplier.

From formulas (282) follows that the linear combination of potentials W^{ν} and $\overset{3}{\mathrm{W}}{}^{\nu}$, identified with electromagnetic potential A^{ν}:

$$\mathrm{A}^{\nu} = p_S \mathrm{W}^{\nu} + p_T \overset{3}{\mathrm{W}}{}^{\nu},$$

is identically zero in the mymino wave.
On seeing the formulas (282), the reader may exclaim in disbelief: can ***it*** really be the neutrino? Perhaps, one has just to get used to this description: there is no other ***classical*** picture for neutrino in physics. Formulas (282) give a complete description of the object in its intrinsic frame of reference. Similarly to the maxwellian neutrino, described in the article [1], mixed singlet-triplet neutrino (282) in the intrinsic frame is the current and field ripple that does not change in time. According to the mathematical model, constructed here, we can give the following description of mymino: ***Maxwell-Yang-Mills neutrino is isotropic current*** N^{ν}***, which couples two potentials*** $\boldsymbol{W}^{\nu}$ ***and*** $\overset{3}{\boldsymbol{W}}{}^{\nu}$ ***from the two sectors of physics- singlet and triplet; the constraint of these three isotropic vectors*** N^{ν}***,*** $\boldsymbol{W}^{\nu}$ ***and*** $\overset{3}{\boldsymbol{W}}{}^{\nu}$ ***is so strong that it "switches off" two other components of YM-triplet of potentials and "switches off" the nonlinearity which is inherent to the triplet sector of physics; this closely coupled triple of vectors generates one physical object - mymino wave***.
Within the framework of the ***classical*** theory, constructed here, it would be incorrect to

speak about the "interaction of neutrino with field Z^ν", although this is a conventional way now to describe some of the weak interaction processes. Indeed, the term of the form $N^\nu Z_\nu$ is present in the mymino Lagrangian (257), which allows to speak about the interaction of current and field, but this is the interaction which ***one*** physical object, not ***two*** objects, distinguishable from each other, corresponds to: field Z^ν is a ***part*** of neutrino rather than its environment.

While calculating the components of singlet $\mathrm{W}^{\mu\nu}$ and triplet $\mathbf{W}^{\mu\nu}$ field tensor, we can find that in the mymino intrinsic frame of reference, the corresponding "electric" fields $\mathrm{W}^{0\nu}$ and $\overset{3}{\mathrm{W}}{}^{0\nu}$ are longitudinal (only the components W^{01} and $\overset{3}{\mathrm{W}}{}^{01}$ are nonzero), and "magnetic" fields are transverse (only components W^{12} and $\overset{3}{\mathrm{W}}{}^{12}$, corresponding to z-component of the magnetic field vectors, are nonzero in the "magnetic" part of the field tensor). Calculation of the corresponding energy – momentum tensor components shows that in the mymino intrinsic frame of reference, the three-dimensional vector of energy flux density is directed along axis y, as well as the three-dimensional current in the wave – that is, orthogonally to the three-dimensional vector of the mymino wave. The three-dimensional velocity of energy flux is equal to the velocity of light.

Undoubtedly, in this phase of construction of physics of currents and fields, the distance between uncommon, but quite clear picture of the classical relativistic neutrino presented here, and, for example, phenomenological Fermi theory of β-decay – quantum and non-relativistic theory, seem almost infinite. However, the same physical object, neutrino, appears in both theories, and the distance between such different portraits of neutrino has to be once overcome.

10.1.3 Non-linear Mymino Model. Restrictions on Pseudo-Euclidean Potential Modules

It is possible to consider a mymino model, which is a little bit more general than model (277). In model (277), all four potentials Z^ν, $\overset{a}{\mathrm{W}}{}^\nu$, appearing in the problem, are isotropic vectors. However, both holonomic (264) and nonholonomic (273) constraints of the problem will be identically satisfied if we choose a more general form of potentials $\overset{a}{\mathrm{W}}{}^\nu$ in the wave intrinsic system, while preserving the form of Z^ν, according to (277):

$$\overset{a}{\mathrm{W}}{}^\nu = \left\{ \overset{a}{\mathrm{T}};\ 0;\ \overset{a}{\mathrm{T}};\ \mathrm{V} \right\};\ a = 1,\ 2,\ 3. \tag{283}$$

For wave form (283) three YM-potentials $\overset{a}{\mathrm{W}}{}^\nu$ have the same z-component in the intrinsic frame of reference, and the same square of pseudo-Euclidean module, which is equal to $-\mathrm{V}^2$. Yang-Mills inertia tensor for wave (283) is determined by the value of V^2:

$$\hat{\mathrm{I}} = \mathrm{V}^2 \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}. \tag{284}$$

Substitution of YM-tensor (284) into field equations (276) will provide the following system of equations for five unknown functions T, $\overset{a}{\mathrm{T}}$, V:

$$\begin{aligned}
&\mathrm{T}'' + k_S^2\mathrm{T} && -p_S p_T^2\,\mathrm{V}^2\left(\overset{1}{\mathrm{T}} + \overset{2}{\mathrm{T}} - 2\,\overset{3}{\mathrm{T}}\right) = 0;\\
&\overset{1}{\mathrm{T}}'' && +p_T^2\,\mathrm{V}^2\left(\overset{2}{\mathrm{T}} + \overset{3}{\mathrm{T}} - 2\,\overset{1}{\mathrm{T}}\right) = 0;\\
&\overset{2}{\mathrm{T}}'' && +p_T^2\,\mathrm{V}^2\left(\overset{3}{\mathrm{T}} + \overset{1}{\mathrm{T}} - 2\,\overset{2}{\mathrm{T}}\right) = 0;\\
&\overset{3}{\mathrm{T}}'' - p_S k_S^2\mathrm{T} && +p_T^2\,\mathrm{V}^2\left(\overset{1}{\mathrm{T}} + \overset{2}{\mathrm{T}} - 2\,\overset{3}{\mathrm{T}}\right) = 0;\\
&\mathrm{V}'' = 0.
\end{aligned} \tag{285}$$

The last equation (285), except for trivial solution V = 0, embedded into the linear mymino model (277), (278), (279), has a linearly growing solution:

$$\mathrm{V} = ax + b. \tag{286}$$

If in (286) $a \neq 0$, constant b in (286) can be attached a zero value by simple shifting of the reference point along axis x.
Let us first consider solution (285) with $a = 0$. This solution per se is of no physical interest, since the state $\mathrm{V} = b = \mathrm{const}$ corresponds to imposition of some external field on mymino wave (277). But this solution allows to see the instability of mymino in such external field.
At $\mathrm{V} = b = \mathrm{const}$, the first four equations of system (285) turn into the system of linear homogeneous equations with a constant coefficient. Trying functions T and $\overset{a}{\mathrm{T}}$ in the form of $\mathrm{const}\cdot \mathrm{e}^{px}$ from (285), we can obtain a characteristic equation relative to eigenvalues of p:

$$p = \pm\mu\sqrt{u+2},$$

where $\mu = p_T b$, and u satisfies the equation:

$$(u-1)\,(u+2)\,\left(u^2 + u\,\left(1+\kappa^2\right) - 2 - \kappa^2\left(1 - 2p_S^2\right)\right) = 0, \tag{287}$$

with $\kappa = k_S/\mu$.
Polynomial root (287) $u_1 = 1$ demonstrates exponential instability of mymino wave: $p_1 = \pm\mu\sqrt{3}$. The second root $u_2 = -2$ generates the possibility of linear growth of instability: $p_2 = 0$ (a double root). The remaining two roots $u_\pm$ of polynomial (287) have the form:

$$u_\pm + 2 = \frac{1}{2}\left(3 - \kappa^2 \pm \left(3+\kappa^2\right)\sqrt{1 - \frac{8\kappa^2 p_S^2}{\left(3+\kappa^2\right)^2}}\right). \tag{288}$$

In the asymptotic behavior of weak external perturbation ($\mu \to 0$; $\kappa \to \infty$), roots (288) describe a mymino wave ($p_- \propto \pm i k_S$), which long-wave oscillations $\left(p_+ \propto \pm i\mu p_S\sqrt{2}\right)$ are imposed on. In the asymptotic behavior of strong external perturbation ($\mu \to \infty$; $\kappa \to 0$),

roots $u_\pm$ "duplicate" roots u_1 and u_2 $\left(p_+ \propto \pm\mu\sqrt{3}; p_- \propto 0\right)$.
This analysis demonstrates that, with $a = 0$ in (286), the components of potentials T and $\overset{a}{\mathrm{T}}$ unrestrictedly grow along the longitudinal wave coordinate, but pseudo-Euclidean modules of 4-potentials $\overset{a}{\mathrm{W}}{}^\nu$ remain constant, and we have no mechanism for suppressing the growth of T and $\overset{a}{\mathrm{T}}$.
The solutions to problem (285), with $\mathrm{V} = ax$, $a \neq 0$, will also grow unrestrictedly along the longitudinal coordinates of the wave. But for such solutions we have a mechanism for suppression T and $\overset{a}{\mathrm{T}}$ growth ***if we decide to extend the introduced earlier restrictions of pseudo-Euclidean module of YM-currents (7) on YM-potentials*** $\overset{a}{\mathrm{W}}{}^\nu$, and assume that

$$\mathrm{V}^2 \leq \mathrm{j}_\mathrm{T}^2. \tag{289}$$

For each value of longitudinal coordinate x, at which the equality sign is reached in (289), we have to change the signs for V', T' and $\overset{a}{\mathrm{T}}{}'$[41]. In accordance with the determination of potential (258), singlet potential W for wave (283) has the form:

$$\mathrm{W}^\nu = \frac{1}{p_T}\left\{\mathrm{T} + p_S\,\overset{3}{\mathrm{T}};\ 0;\ \mathrm{T} + p_S\,\overset{3}{\mathrm{T}};\ p_S\mathrm{V}\right\},$$

and, correspondingly, its pseudo-Euclidean module is determined by value V:

$$\mathrm{W}^\nu\mathrm{W}_\nu = -\frac{p_S^2}{p_T^2}\mathrm{V}^2. \tag{290}$$

Assuming that triplet potentials, as well as triplet currents, have restricted pseudo-Euclidean module (283), following the same logic, we have to assume that singlet potential W^ν also has module restriction – the same as singlet current (96):

$$-\mathrm{W}^\nu\mathrm{W}_\nu \leq \mathrm{j}_S^2. \tag{291}$$

Comparison of (289), (290) and (291) allows to relate the limiting values of currents j_S and j_T:

$$\mathrm{j}_S\, p_\mathrm{T} = p_S \mathrm{j}_\mathrm{T},$$

or

$$\mathrm{j}_\mathrm{T} = \mathrm{tg}\theta_w \cdot \mathrm{j}_S, \tag{292}$$

where θ_w is Weinberg angle.
It should be noted that in nonlinear mymino wave (285), in contrast to linear wave (282), electromagnetic potential A^ν is nonzero.

[41] If you do not like this appalling violent procedure, which restricts the growth of solutions in mymino wave, you have to come up with some other procedure – perhaps no less appalling – or even abandon the attempt to construct the classical theory of neutrino and, the classical version of singlet-triplet physics, on the whole.

10.2 Mixed Singlet-triplet Two-current One-neutrino State

10.2.1 The Lagrangian and Field Equations

Let us consider such a mixed state, where along with the singlet-triplet neutrino current N^ν there is one more YM-triplet current, for example, $\overset{1}{\mathrm{J}}{}^\nu$:

$$\mathrm{J}^\nu = N^\nu;\ \overset{3}{\mathrm{J}}{}^\nu = -N^\nu;\ N^\nu N_\nu = 0;\ \overset{2}{\mathrm{J}}{}^\nu = 0;\ \overset{1}{\mathrm{J}}{}^\nu = \mathrm{j}^\nu;\ \mathrm{j}^\nu \mathrm{j}_\nu < 0. \tag{293}$$

The state Lagrangian (293), in accordance with general form of the basic Lagrangian of the singlet-triplet theory (17), can be written in the following form:

$$L = -\frac{1}{8p_T^2}\mathrm{j}^\nu \mathrm{j}_\nu - \frac{1}{2p_T p_S} N^\nu Z_\nu - \frac{1}{2p_T}\mathrm{j}^\nu \overset{1}{\mathrm{W}}_\nu - \frac{1}{16\pi}\left(\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} + \mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}\right) - \frac{\lambda}{4p_T p_S} N^\nu N_\nu. \tag{294}$$

The last term in the Lagrangian (294) is a "penalty" for neutrino character of current N^ν; λ is the Lagranges multiplier; Z^ν is the linear combination of potentials W and $\overset{3}{\mathrm{W}}{}^\nu$ (258).

The following field equations result from the Lagrangian form (294):

$$\lambda N^\nu + Z^\nu = 0;$$

$$\mathrm{j}^\nu + 2p_T \overset{1}{\mathrm{W}}{}^\nu = 0; \quad (\text{"current equations"}) \tag{295}$$

$$\partial_\mu \mathrm{W}^{\mu\nu} = \frac{2\pi}{p_S} N^\nu; \quad (\text{"Maxwell equations"}) \tag{296}$$

$$\partial_\mu \mathbf{W}^{\mu\nu} + p_T\, \mathbf{W}_\mu \times \mathbf{W}^{\mu\nu} = \frac{2\pi}{p_T}\left(\mathrm{j}^\nu \overset{1}{\mathbf{e}} - N^\nu \overset{3}{\mathbf{e}}\right); \quad (\text{"Yang-Mills equations"}) \tag{297}$$

In equations (297) $\overset{1}{\mathbf{e}}$ and $\overset{3}{\mathbf{e}}$ are the corresponding YM-unit vectors with YM-components $\overset{1}{\mathbf{e}} = \{1;\, 0;\, 0\}$; $\overset{3}{\mathbf{e}} = \{0;\, 0;\, 1\}$.

From current equations (295) follows that the combined potential Z^ν, similarly to current N^ν, is isotropic:

$$Z^\nu Z_\nu = 0. \tag{298}$$

The gauge condition by divergence of all potentials can be imposed on the problem under consideration:

$$\begin{aligned} &\partial_\nu Z^\nu = 0;\\ &\partial_\nu \mathbf{W}^\nu = 0. \end{aligned} \tag{299}$$

The algebraic relations, which the solution of the field equations (295), (296), (297) must obey, follow from the differential condition for YM-triplet currents (19) and current equations (295):

$$\begin{aligned} &Z^\nu \overset{2}{\mathrm{W}}{}^\nu = 0;\\ &\overset{1}{\mathrm{W}}{}^\nu \overset{2}{\mathrm{W}}_\nu = 0;\\ &\overset{1}{\mathrm{W}}_\nu \left(2p_T \overset{3}{\mathrm{W}}{}^\nu - \mathrm{q} Z^\nu\right) = 0. \end{aligned} \tag{300}$$

In the last equation (300) it is indicated that $q = -1/_\lambda$.
Relations (298) and (300) specify the system of holonomic constraints, imposed on the solution of the field equations of the problem.
Extracting currents j^ν, N^ν and singlet potential W^ν through current equations, we can reduce the problem under consideration to the system of wave equations relative to four potentials Z^ν and $\mathbf{W}^\nu$:

$$\begin{aligned} &\Box Z^\nu + k_S^2 Z^\nu + p_S p_T^2 \overset{3b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu = -p_S p_T \overset{3}{\mathrm{h}}{}^\nu; \\ &\Box \mathbf{W}^\nu - \left(4\pi \overset{1}{\mathbf{e}}\overset{1}{\mathrm{W}}{}^\nu + k_S^2 p_S Z^\nu \overset{3}{\mathbf{e}}\right) - p_T^2 \,\hat{\mathrm{I}}\mathbf{W} = p_T \mathbf{h}. \end{aligned} \qquad (301)$$

In the equations (301), parameter k_S is determined by formula (281), and vector $\mathbf{h}^\nu$ – by relation (266). Nonholonomic constraint (273) is imposed on YM-potentials $\mathbf{W}^\nu$.

10.2.2 Solution in the Form of a Plane Wave

Let us consider the solution of the problem (301) in the form of a plane wave. In such solution potentials Z^ν and $\mathbf{W}^\nu$ depend only on a single argument, the wave phase $\phi = k^\nu x_\nu$. Since in the problem (301) there is isotropic vector Z^ν, wave vector k^ν can only be space-like. In the wave intrinsic system, wave vector k^ν has only one nonzero component in longitudinal coordinate x. D'Alembert operator in (301) is reduced to the operator of the twofold differentiation in the longitudinal coordinate. Vector $\mathbf{h}^\nu$ in the plane wave vanishes. Field equations (301) turn into the system of ordinary differential equations:

$$\begin{aligned} &Z^{\nu\,\prime\prime} + k_S^2 Z^\nu && + p_S p_T^2 \overset{3b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu = 0; \\ &\overset{1}{\mathrm{W}}{}^{\nu\,\prime\prime} - 4\pi \overset{1}{\mathrm{W}}{}^\nu && - p_T^2 \overset{1b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu = 0; \\ &\overset{2}{\mathrm{W}}{}^{\nu\,\prime\prime} && - p_T^2 \overset{2b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu = 0; \\ &\overset{3}{\mathrm{W}}{}^{\nu\,\prime\prime} - k_S^2 p_S Z^\nu && - p_T^2 \overset{3b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu = 0. \end{aligned} \qquad (302)$$

In equations (302), the prime stands for longitudinal coordinate x derivative.
Differential conditions (299), as well as in other problems of the waves with space-like wave vector, mean that no potential has a longitudinal component:

$$Z^1 = 0;\ \overset{a}{\mathrm{W}}{}^1 = 0;\ a = \overline{1,\,3}. \qquad (303)$$

Considering the wave with linear polarization, we can assume, without restricting generality, that isotropic vector Z^ν in the wave intrinsic system has the form:

$$Z^\nu = \mathrm{T}\,\{1;\ 0;\ 1;\ 0\}\,, \qquad (304)$$

where T is some desired function of longitudinal coordinate x.
Both holonomic and nonholonomic constraints, imposed on solution (302), will be satisfied

if we try YM-potentials $\overset{a}{\mathrm{W}}{}^{\nu}$ in the following form:

$$\begin{aligned} \overset{1}{\mathrm{W}}{}^{\nu} &= \left\{ \overset{1}{\mathrm{T}};\, 0;\, \overset{1}{\mathrm{T}};\, \mathrm{V} \right\}, \\ \overset{2}{\mathrm{W}}{}^{\nu} &= \overset{2}{\mathrm{T}} \left\{1;\, 0;\, 1;\, 0\right\}, \\ \overset{3}{\mathrm{W}}{}^{\nu} &= \overset{3}{\mathrm{T}} \left\{1;\, 0;\, 1;\, 0\right\}. \end{aligned} \tag{305}$$

In five-component solution (304), (305), functions T, $\overset{1}{\mathrm{T}}$, $\overset{2}{\mathrm{T}}$, $\overset{3}{\mathrm{T}}$ and V are desired ones. The choice of five-component structure of the solution is not the only possible method to satisfy all the constraint equations. However, this partial choice of the solution structure allows seeing the fundamental problem of unrestricted growth of the solutions to equations (302).

YM-inertia tensor $\hat{\mathrm{I}}$ for wave (305) has the form:

$$\hat{\mathrm{I}} = \mathrm{V}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{306}$$

Substitution of tensor (306) into field equations (302) for wave (304), (305) gives the following system of differential equations:

$$\begin{aligned} &\mathrm{V}'' - 4\pi \mathrm{V} &&= 0, \\ &\overset{1}{\mathrm{T}}{}'' - 4\pi\, \overset{1}{\mathrm{T}} &&= 0, \\ &\overset{2}{\mathrm{T}}{}'' - p_T^2\, \mathrm{V}^2\, \overset{2}{\mathrm{T}} &&= 0, \\ &\overset{3}{\mathrm{T}}{}'' - p_T^2\, \overset{3}{\mathrm{T}} - k_S^2 p_S \mathrm{T} &&= 0, \\ &\mathrm{T}'' + k_S^2 \mathrm{T} + p_S p_T^2\, \mathrm{V}^2\, \overset{3}{\mathrm{T}} &&= 0. \end{aligned} \tag{307}$$

The possible, but uninteresting situation of choice of the trivial solutions to the first three equations of system (307) is: $\mathrm{V} = 0$; $\overset{1}{\mathrm{T}} = 0$, $\overset{2}{\mathrm{T}} = 0$, returns us to the mymino wave considered above. Except for this degenerate situation, system (307) has no solutions restricted at the growing of longitudinal coordinate x. The first two equations of the system (307), linear and independent from the rest of the equations (307), have solutions, exponentially growing with growth of x. Substitution of these growing solutions as coefficients of other equations of the system generates unrestricted growth of the rest of the problem variables (307).

10.2.3 Two Methods of Allowing for Current Restriction. Neutrino Instability

Is it possible to cope with this non-physical solution growth?

It is possible to take into account the restriction on pseudo-Euclidean module of current

j^{ν} (7). From current equations (295) and (305) follows that

$$\mathrm{j}^{\nu}\,\mathrm{j}_{\nu} = 4p_T^2\ \overset{1}{\mathrm{W}}{}^{\nu}\ \overset{1}{\mathrm{W}}_{\nu} = -4p_T^2\,\mathrm{V}^2,$$

consequently,

$$\mathrm{V}^2 \leq \left(\frac{\mathrm{j}_T}{2p_T}\right)^2. \tag{308}$$

The growing solution of the system (307) is applicable only for such values of longitudinal coordinate x while inequality (308) is satisfied.

At obtaining the equality in (308) with some $x = x_e$, we have to reverse the signs of all the derivatives of potentials V', T', $\overset{1}{\mathrm{T}}{}'$, $\overset{2}{\mathrm{T}}{}'$, $\overset{3}{\mathrm{T}}{}'$, while maintaining the continuity of the potentials themselves and to continue the solution further along the longitudinal coordinate x, with $x > x_e$ under the new initial conditions. This barbaric method generates a restricted solution with discontinuities of derivatives in the form of some ripple of potentials and currents, frozen in the intrinsic system of the wave. For correct numerical description of this ripple, the curvature of space, caused by the wave itself (307), should be taken into account.

The second, and probably, the deeper interpretation of inequality (308) consists of refusal of the plane wave picture, covering the whole space-time. At obtaining the equality in (308), the plane wave vanishes, the formation of some cavitated latebrae zones, not containing current j^{ν}, takes place, and then the formation of the pomerium's system, specifying the outer boundaries of current zones, takes place. In other words, the two-current wave turns into one or a few massive triplet particles, described in part 9.3.6, surrounded by neutrino fields. However, it would be naive to expect that such an intimate and, undoubtedly, quantum process of massive particle's production permits the adequate numerical description in terms of the classical Lagrangians.

Comparison of the equations, which describe mymino neutrino wave (10.1) with the system of equations (307), describing the two-current singlet-triplet wave, allows to make the following assumption.

Mymino neutrino wave (282), stable in itself, is unstable against the buildup of the two-current state: no matter how small initial perturbations are (i.e. no matter how small "inoculating" values of current $\overset{1}{\mathrm{J}}{}^{\nu}$ components are), the solution of the neutrino problem turns fast into the solution of two-current singlet-triplet problem due to the exponential growth of $\overset{1}{\mathrm{T}}$ and V, but this two-current wave without latens and pomerium quickly turns into a massive YM-particle having inner latebrae voids, inner latens boundaries and outer pomerium boundary. Within the framework of this picture, the neutrino production in any process must be accompanied by the production of a massive YM-particle with a very small time delay (probably, with delay of about $r_0/_c \propto 10^{-36}$ seconds)[42].

[42] According to beliefs common for modern physics, such massive "satellite" of neutrino is W-boson. We do not know whether identification of W-boson with massive one-current particle is acceptable (p.9.3.6.) W-boson is treated as massive quantum of free (zero-current) YM-field. The particle described in p.

10.3 Mixed Singlet-triplet Three-current One-neutrino State

10.3.1 The Lagrangian and Field Equations

Let us turn to the study of such a mixed state, where, besides neutrino singlet-triplet current N^ν, there are two more YM-currents:

$$\mathrm{J}^\nu = N^\nu;\ \overset{3}{\mathrm{J}}{}^\nu = -N^\nu;\ N^\nu N_\nu = 0;\ \overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu < 0;\ \overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu < 0;\ \overset{1}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu = 0. \quad (309)$$

The last equality in the description of this state (309) results from current $\overset{3}{\mathrm{J}}{}^\nu$ isotropy and the condition of the inner normalization of YM-triplet currents (32).
The Lagrangian of the state (309), in accordance with the type of base Lagrangian of ST-theory (17), can be denoted in the form:

$$\begin{aligned} L = &-\frac{1}{8p_T^2}\left(\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{J}}_\nu + \overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{J}}_\nu\right) - \frac{1}{2p_T}\left(\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathrm{W}}_\nu + \overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathrm{W}}_\nu\right) - \frac{1}{2p_T p_S} N^\nu Z_\nu - \\ &-\frac{1}{16\pi}\left(\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} + \mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}\right) - \frac{\lambda}{4p_T p_S} N^\nu N_\nu. \end{aligned} \quad (310)$$

In (310), Z^ν is the linear combination of potentials W^ν and $\overset{3}{\mathrm{W}}{}^\nu$ (258) introduced before; λ is the Lagranges multiplier. The last term in (310) is the "penalty" for the current's neutrino character. Besides this "penalty", the Lagrangian (310) must contain one more term – "penalty" for currents $\overset{1}{\mathrm{J}}{}^\nu$ and $\overset{2}{\mathrm{J}}{}^\nu$ orthogonality. For the reasons expressed above, in p. 9.4.1, we do not include this penalty into the Lagrangian (310), supposing that the condition of currents $\overset{1}{\mathrm{J}}{}^\nu$ and $\overset{2}{\mathrm{J}}{}^\nu$ orthogonality is "natural" for this problem.
Field equations, following from the Lagrangian (310), have the following form:

$$\begin{aligned} &\lambda N^\nu + Z^\nu = 0; \\ &\overset{1}{\mathrm{J}}{}^\nu + 2p_T \overset{1}{\mathrm{W}}{}^\nu = 0; \\ &\overset{2}{\mathrm{J}}{}^\nu + 2p_T \overset{2}{\mathrm{W}}{}^\nu = 0; \quad (\text{"current equations"}) \end{aligned} \quad (311)$$

$$\begin{aligned} &\partial_\mu \mathrm{W}^{\mu\nu} = \frac{2\pi}{p_S} N^\nu; \quad (\text{"Maxwell equations"}) \\ &\partial_\mu \mathbf{W}^{\mu\nu} + p_T\, \mathbf{W}_\mu \times \mathbf{W}^{\mu\nu} = \frac{2\pi}{p_T}\left(\overset{1}{\mathrm{J}}{}^\nu \overset{1}{\mathbf{e}} + \overset{2}{\mathrm{J}}{}^\nu \overset{2}{\mathbf{e}} - N^\nu \overset{3}{\mathbf{e}}\right). (\text{"Yang-Mills equations"}) \end{aligned} \quad (312)$$

In equations (312) $\overset{a}{\mathbf{e}}$ is YM-unit vectors.
From current equations (311) follows that the mixed potential Z^ν is isotropic, and YM-potentials $\overset{1}{\mathrm{W}}{}^\nu$ and $\overset{2}{\mathrm{W}}{}^\nu$ are orthogonal to each other (under the orthogonality of the corresponding currents (309)):

$$\begin{aligned} &Z^\nu Z_\nu = 0; \\ &\overset{1}{\mathrm{W}}{}^\nu \overset{2}{\mathrm{W}}_\nu = 0. \end{aligned} \quad (313)$$

9.3.6, consists of mixture of YM-current and YM-field, similarly to massive leptons, which are a mixture of singlet current and singlet field.

Differential conditions for the currents of YM-triplet (19) and current equations (311) generate algebraic conditions for potentials Z^ν and $\overset{a}{\mathrm{W}}{}^\nu$:

$$\begin{aligned} \overset{1}{\mathrm{W}}_\nu \left(2p_T \overset{3}{\mathrm{W}}{}^\nu - \mathrm{q} Z^\nu\right) = 0; \\ \overset{2}{\mathrm{W}}_\nu \left(2p_T \overset{3}{\mathrm{W}}{}^\nu - \mathrm{q} Z^\nu\right) = 0. \end{aligned} \tag{314}$$

In equation (314) it is denoted that $\mathrm{q} = -1/\lambda$.
Relations (313) and (314) are holonomic constraints, which the solution of the field equations (312) obeys.
Extraction of currents and singlet potential from field equations (312) leads to the following system of wave equations relative to potentials Z^ν and $\overset{a}{\mathrm{W}}{}^\nu$:

$$\begin{aligned} &\Box Z^\nu + k_S^2 Z^\nu + p_S p_T^2 \overset{3b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu = 0; \\ &\Box \mathbf{W}^\nu - 4\pi \left(\overset{1}{\mathrm{W}}{}^\nu \overset{1}{\mathbf{e}} + \overset{2}{\mathrm{W}}{}^\nu \overset{2}{\mathbf{e}}\right) - k_S^2 p_S Z^\nu \overset{3}{\mathbf{e}} - p_T^2 \hat{\mathrm{I}} \mathbf{W}^\nu = p_T \mathbf{h}^\nu. \end{aligned} \tag{315}$$

In system (315), describing the three-current state, all notations are the same as in the similar system of equations (301), describing the two-current state.

10.3.2 Solution in the Form of a Plane Wave

For plane wave $\mathbf{h}^\nu \equiv 0$, all variables of the problem depend on longitudinal coordinate x, and wave equations take the form which is similar to equations (302), introduced for the two-current problem:

$$\begin{aligned} Z^{\nu\,\prime\prime} + k_S^2 Z^\nu + p_S p_T^2 \overset{3b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu &= 0; \\ \overset{1}{\mathrm{W}}{}^{\nu\,\prime\prime} - 4\pi \overset{1}{\mathrm{W}}{}^\nu - p_T^2 \overset{1b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu &= 0; \\ \overset{2}{\mathrm{W}}{}^{\nu\,\prime\prime} - 4\pi \overset{2}{\mathrm{W}}{}^\nu - p_T^2 \overset{2b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu &= 0; \\ \overset{3}{\mathrm{W}}{}^{\nu\,\prime\prime} - k_S^2 p_S Z^\nu - p_T^2 \overset{3b}{\mathrm{I}}\overset{b}{\mathrm{W}}{}^\nu &= 0. \end{aligned} \tag{316}$$

All the potentials of Z^ν and $\overset{a}{\mathrm{W}}{}^\nu$, have zero longitudinal potential Z^1 and $\overset{a}{\mathrm{W}}{}^1$.
Solution to problem (316) has to satisfy the four holonomic constraints (313), (314) and the three nonholonomic constraints (273). We do not know whether there are any solutions to the system of differential equations (316), burdened with so many constraints, anyway, we cannot construct a solution of some simple structure, which would automatically account all the constraint equations. If solutions do exist, they grow unrestrictedly along the wave direction. Taking into account the restriction on the growth of pseudo-Euclidean current module, as described in p. 10.2.2, it is possible to construct restricted solutions having discontinuities of the variables.

10.4 Mixed Singlet-triplet Three-current Neutrinoless State (the left-hand three-current state)

10.4.1 The Lagrangian of the Left-hand Three-current State

Let us consider such a mixed triplet-singlet state, where the same space-like current l^ν ($l^\nu l_\nu < 0$) is present both as singlet current J and the third component of YM-triplet of currents $\overset{3}{\mathrm{J}}{}^\nu$:

$$\mathrm{J}^\nu = l^\nu ;\ \overset{3}{\mathrm{J}}{}^\nu = l^\nu . \tag{317}$$

This current state is compatible with formulas of current decomposition (22). Within the framework of this decomposition, current l^ν is the "left-hand" current. The right-hand current r^ν and neutrino current N^ν are missing from the state (317). The algebraic conditions of coupling of currents J^ν and $\overset{3}{\mathrm{J}}{}^\nu$ (20) in state (317) turn into identity.

In state (317) two other YM-currents, $\overset{1}{\mathrm{J}}{}^\nu$ and $\overset{2}{\mathrm{J}}{}^\nu$, must be present, since, due to the conditions of the inner normalization of YM-triplet currents (32), their scalar product is nonzero:

$$2\,\overset{1}{\mathrm{J}}{}^\nu\,\overset{2}{\mathrm{J}}{}_\nu = l^\nu l_\nu . \tag{318}$$

The substitution of formulas (317) into the base Lagrangian of the ST-theory (17) allows us to write the Lagrangian of the left-hand three-current states in the following form:

$$L = L_{cur} + L_{\mathrm{int}} + L_f , \tag{319}$$

where the current Lagrangian L_{cur} contains pseudo-Euclidean squares of all three currents of state:

$$L_{cur} = -\frac{l^\nu l_\nu}{8p_S p_T} - \frac{1}{8p_T^2}\left(\overset{1}{\mathrm{J}}{}^\nu\,\overset{1}{\mathrm{J}}{}_\nu + \overset{2}{\mathrm{J}}{}^\nu\,\overset{2}{\mathrm{J}}{}_\nu\right). \tag{320}$$

The interaction Lagrangian L_{int} is convenient to be written as follows:

$$L_{\mathrm{int}} = -\frac{1}{2p_S p_T} l^\nu Q_\nu - \frac{1}{2p_T}\left(\overset{1}{\mathrm{J}}{}^\nu\,\overset{1}{\mathrm{W}}{}_\nu + \overset{2}{\mathrm{J}}{}^\nu\,\overset{2}{\mathrm{W}}{}_\nu\right), \tag{321}$$

where Q_ν is the linear combination of potentials W^ν and $\overset{3}{\mathrm{W}}{}^\nu$:

$$Q^\nu = p_T \mathrm{W}^\nu + p_S\,\overset{3}{\mathrm{W}}{}^\nu .$$

The field Lagrangian L_f in (319) has an ordinary form (13).

We have to note that usually in Weinberg-Salam theory it is not potential Q^ν itself that is used, but its decomposition into electromagnetic potential A^ν $\left(\mathrm{A}^\nu = p_T \mathrm{W}^\nu + p_S\,\overset{3}{\mathrm{W}}{}^\nu\right)$ and "neutral" potential Z^ν $\left(Z^\nu = p_T \mathrm{W}^\nu - p_S\,\overset{3}{\mathrm{W}}{}^\nu\right)$:

$$\begin{gathered} Q^\nu = 2p_S p_T \mathrm{A}^\nu - \left(p_S^2 - p_T^2\right) Z^\nu , \\ \text{or } Q^\nu = \sin 2\vartheta_W \cdot \mathrm{A}^\nu - \cos 2\vartheta_W \cdot Z^\nu . \end{gathered} \tag{322}$$

If we use decomposition (322), the interaction Lagrangian (321) can be denoted as:

$$L_{\text{int}} = -l^{\nu}\mathrm{A}_{\nu} + \mathrm{ctg}2\vartheta_W \cdot l^{\nu}Z_{\nu} - \frac{1}{2p_T}\left(\overset{1}{\mathrm{J}}{}^{\nu}\,\overset{1}{\mathrm{W}}_{\nu} + \overset{2}{\mathrm{J}}{}^{\nu}\,\overset{2}{\mathrm{W}}_{\nu}\right), \tag{323}$$

Expression (323) allows us to speak, using the generally accepted "jargon" of theoretical physics, about the fact that *the left-hand current* l^{ν} *"interacts" with electromagnetic potential* A^{ν} *(the first term in (323)), and "interacts" with neutral field* Z^{ν} *(the second term in (323)).* At full formal equality of (321) and (323), formula (321) seems more convenient and compact: the interaction Lagrangian (321) contains only one scalar product, which includes current and combined potential Q^{ν} – it is the one that the left-hand current l^{ν} "interacts" with[43].

While recording the three-current state Lagrangian (319), it is convenient to introduce rescaled currents, assuming that

$$\begin{aligned} m^{\nu} &= \frac{1}{2p_S p_T} l^{\nu}; \\ \overset{1}{\mathrm{j}}{}^{\nu} &= \frac{1}{2p_T}\,\overset{1}{\mathrm{J}}{}^{\nu};\ \overset{2}{\mathrm{j}}{}^{\nu} = \frac{1}{2p_T}\,\overset{2}{\mathrm{J}}{}^{\nu}, \end{aligned} \tag{324}$$

and also the conventional YM-triplet of currents $\mathbf{j}^{\nu}$ with zero third YM-component:

$$\mathbf{j}^{\nu} = \left\{\overset{1}{\mathrm{j}}{}^{\nu};\ \overset{2}{\mathrm{j}}{}^{\nu};\ 0\right\}. \tag{325}$$

In notations (324), (325), the three-current state Lagrangian takes the form which is compact enough:

$$L = -\frac{1}{2}m^{\nu}m_{\nu} - \frac{1}{2}\mathbf{j}^{\nu}\mathbf{j}_{\nu} - m^{\nu}Q^{\nu} - \mathbf{j}^{\nu}\cdot\mathbf{W}_{\nu} - \frac{1}{16\pi}\left(\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} + \mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}\right). \tag{326}$$

Weinberg coefficients p_S and p_T, which are missing from the Lagrangian (326) an explicit form, of course, cannot be excluded from the description of mixed singlet-triplet state by any scale transformation. They remain in the determination of potential Q^{ν} (322), in the expression for YM-field tensor $\mathbf{W}^{\mu\nu}$ (15) and in normalizing condition (318), which has the following form in the notations (324):

$$2\,\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}_{\nu} = p_S^2\, m^{\nu}m_{\nu}. \tag{327}$$

[43] Acknowledging the acceptability of using the stable term "interaction Lagrangian", rooted due to historical reasons, we tend to consider the term "interaction" (of currents and fields) itself, as an archaic and inaccurate one. In three-current state, current l^{ν} and potential Q^{ν}, while interweaving, form an integrated physical object – a "three-current state", and it would be incorrect to speak about "interaction" of current and field – as well as, by reasons, cited in p. 10.1.2, it is incorrect to speak about interaction of neutrino current N^{ν} and neutral potential Z^{ν}, which form an integrated physical object – Maxwell-Yang-Mills neutrino. The term "interaction" is appropriate only as a term that allows to provide approximate description of quasi-stationary states that have two or more current zones where each of them has its own pomerium, and these zones are located far enough from each other.

Condition (327) is a holonomic constraint, imposed on functional arguments of the Lagrangian (326). Generally speaking, accounting of this constraint requires additional term L_{ad} for the base Lagrangian of the three-current problem (326):

$$L_{ad} = \eta\left(2\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathrm{j}}_{\nu} - p_S^2\, m^{\nu} m_{\nu}\right), \tag{328}$$

with undetermined Lagrange's multiplier η.

By introducing additional term (328) into the Lagrangian, with $\eta \neq 0$, we actually introduce some "Higgs-like" field η which stimulates us to satisfy the condition (327).

It is reasonable to try to construct a theory of three-current states on the basis of the Lagrangian (326), supposing that condition (327) is "natural", i.e. can be also satisfied with $\eta = 0$. If non-trivial solutions to the three-current problem do not exist with $\eta = 0$, three-current states may exist only as non-stationary transition states between other states.

10.4.2 Field Equations of the Left-hand Three-current State

The independent functional arguments of the Lagrangian are currents m^{ν} and $\mathbf{j}^{\nu}$, and potentials W^{ν} and $\mathbf{W}^{\nu}$. By varying the Lagrangian (326) by these arguments, we obtain field equations of the left-hand three-current state:

$$\begin{aligned} & m^{\nu} + Q^{\nu} = 0; \\ & \overset{1}{\mathrm{j}}{}^{\nu} + \overset{1}{\mathrm{W}}{}^{\nu} = 0; \\ & \overset{2}{\mathrm{j}}{}^{\nu} + \overset{2}{\mathrm{W}}{}^{\nu} = 0. \quad \text{("current equations")} \end{aligned} \tag{329}$$

$$\partial_{\mu}\mathrm{W}^{\mu\nu} = 4\pi p_T m^{\nu}. \quad \text{("Maxwell equations")} \tag{330}$$

$$\partial_{\mu}\mathbf{W}^{\mu\nu} + p_T\,\mathbf{W}_{\mu}\times\mathbf{W}^{\mu\nu} = 4\pi\left(\overset{1}{\mathrm{j}}{}^{\nu}\,\overset{1}{\mathbf{e}} + \overset{2}{\mathrm{j}}{}^{\nu}\,\overset{2}{\mathbf{e}} + p_S m^{\nu}\,\overset{3}{\mathbf{e}}\right). \text{("Yang-Mills equations")} \tag{331}$$

General gauge conditions by 4-divergence of potentials should be added to field equations (329), (330), (331):

$$\partial_{\mu}\mathbf{W}^{\mu} = 0;\ \partial_{\mu}\mathrm{W}^{\mu} = 0. \tag{332}$$

Conditions (332) are combined with the conditions for conservation of all the three currents of the problem:

$$\partial_{\nu} m^{\nu} = 0;\ \partial_{\nu}\,\overset{1}{\mathrm{j}}{}^{\nu} = 0;\ \partial_{\nu}\,\overset{2}{\mathrm{j}}{}^{\nu} = 0. \tag{333}$$

and, in their turn, conditions (333), along with general differential condition for triplet current, generate the system of holonomic constraints imposed on the solution of the field equations of the left-hand three-current state:

$$\begin{aligned} & p_S m^{\nu}\,\overset{1}{\mathrm{W}}_{\nu} = \overset{1}{\mathrm{j}}{}^{\nu}\,\overset{3}{\mathrm{W}}_{\nu}; \\ & p_S m^{\nu}\,\overset{2}{\mathrm{W}}_{\nu} = \overset{2}{\mathrm{j}}{}^{\nu}\,\overset{3}{\mathrm{W}}_{\nu}; \\ & \overset{1}{\mathrm{W}}_{\nu}\overset{2}{\mathrm{j}}{}^{\nu} = \overset{2}{\mathrm{W}}_{\nu}\overset{1}{\mathrm{j}}{}^{\nu}. \end{aligned} \tag{334}$$

As a result of adding currents coupling (327) to expressions (334), we obtain a complete system of equations describing the left-hand three-current state under consideration in terms of currents and potentials. Exclusion of currents and singlet potential W^ν from this system of equations results in more compact description containing only four potentials $\overset{a}{\mathrm{W}}_\nu$ and Q^ν:

$$\begin{aligned}
&-\Box Q^\nu + 4\pi Q^\nu + p_S p_T \overset{3}{\mathrm{h}}{}^\nu + p_S p_T^2 \overset{3b}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu = 0;\\
&-\Box \overset{1}{\mathrm{W}}{}^\nu + 4\pi \overset{1}{\mathrm{W}}{}^\nu + p_T \overset{1}{\mathrm{h}}{}^\nu + p_T^2 \overset{1b}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu = 0;\\
&-\Box \overset{2}{\mathrm{W}}{}^\nu + 4\pi \overset{2}{\mathrm{W}}{}^\nu + p_T \overset{2}{\mathrm{h}}{}^\nu + p_T^2 \overset{2b}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu = 0;\\
&-\Box \overset{3}{\mathrm{W}}{}^\nu + 4\pi p_S Q^\nu + p_T \overset{3}{\mathrm{h}}{}^\nu + p_T^2 \overset{3b}{\mathrm{I}} \overset{b}{\mathrm{W}}{}^\nu = 0,
\end{aligned} \tag{335}$$

where vector $\mathbf{h}^\nu$, introduced by formula (266), vanishes for plane waves.
The system of four field equations (335) is supplemented by conditions (332) and holonomic constraints (327) and (334). With regard to current equations (329), one of equations (334) transforms into an identity, and the remaining constraint equations take the form:

$$\begin{aligned}
&2 \overset{1}{\mathrm{W}}{}^\nu \overset{2}{\mathrm{W}}_\nu = p_S^2 Q^\nu Q_\nu;\\
&\overset{1}{\mathrm{W}}{}^\nu \mathrm{R}_\nu = 0;\\
&\overset{2}{\mathrm{W}}{}^\nu \mathrm{R}_\nu = 0;
\end{aligned}$$

where

$$\mathrm{R}_\nu = p_S Q_\nu - \overset{3}{\mathrm{W}}_\nu \,. \tag{336}$$

10.4.3 Left-hand Three-current state as a plane wave

Solution to the system of wave equations (335) can be tried in the form of a plane wave with wave vector $k^\nu = \{\omega; \mathbf{k}\}$. If vector k^ν is time-like $\left(\omega^2 > \mathbf{k}^2\right)$, in the intrinsic wave system $\mathbf{k} = 0$ and all potentials and currents in this system depend only on time, and according to (332) time components of potentials vanish. Let us introduce the conventional three-dimensional notations for potentials in the intrinsic system of the plane wave:

$$Q^\nu = \{0; \mathbf{U}\}\,; \; \overset{a}{\mathrm{W}}{}^\nu = \left\{0; \overset{a}{\mathbf{U}}\right\}. \tag{337}$$

In notations (337), wave equations (335) in the intrinsic wave system take the form of a system of four ordinary differential equations:

$$\begin{aligned}
&\mathbf{U}^{\bullet\bullet} + 4\pi \mathbf{U} + p_S p_T^2 \overset{3b}{\mathrm{I}} \overset{b}{\mathbf{U}} = 0;\\
&\overset{1}{\mathbf{U}}{}^{\bullet\bullet} + 4\pi \overset{1}{\mathbf{U}} + p_T^2 \overset{1b}{\mathrm{I}} \overset{b}{\mathbf{U}} = 0;\\
&\overset{2}{\mathbf{U}}{}^{\bullet\bullet} + 4\pi \overset{2}{\mathbf{U}} + p_T^2 \overset{2b}{\mathrm{I}} \overset{b}{\mathbf{U}} = 0;\\
&\overset{3}{\mathbf{U}}{}^{\bullet\bullet} + 4\pi p_S \mathbf{U} + p_T^2 \overset{3b}{\mathrm{I}} \overset{b}{\mathbf{U}} = 0.
\end{aligned} \tag{338}$$

In these equations (338), the dot stands for time derivative in the wave intrinsic system and, as usual:

$$\overset{ab}{\mathrm{I}} = -\overset{a}{\mathbf{U}} \cdot \overset{b}{\mathbf{U}} + \overset{ab}{\delta} \overset{c}{\mathbf{U}} \cdot \overset{c}{\mathbf{U}}, \tag{339}$$

Equations of holonomic constraints (335) in three-dimensional notations (337) take the form:

$$\begin{aligned} &2\ \overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}} = p_S^2\, \mathbf{U}^2; \\ &\overset{1}{\mathbf{U}} \cdot \mathbf{R} = 0; \\ &\overset{2}{\mathbf{U}} \cdot \mathbf{R} = 0; \end{aligned} \tag{340}$$

where

$$\mathbf{R} = p_S\, \mathbf{U} - \overset{3}{\mathbf{U}}\, . \tag{341}$$

Nonholonomic constraints (338) are imposed on solution of the system:

$$\overset{abc}{\varepsilon} \overset{b}{\mathbf{U}} \cdot \overset{c}{\mathbf{U}}{}^{\bullet} = 0. \tag{342}$$

Coupling equations (342), as well as in other problems studied above, are the restrictions imposed on the initial conditions of the system (338).
Let us use the term *lethos* (left-hand threecurrent oscillator) to denote the object obeying the system of differential equations (338), holonomic constraints (340) and nonholonomic constraints (342). We do not have a theorem which would guarantee the existence of the object lethos.
It is possible to demonstrate two partial solutions to this problem:
1. If we suppose that

$$\sqrt{2}\ \overset{1}{\mathbf{U}} = \sqrt{2}\ \overset{2}{\mathbf{U}} = \overset{3}{\mathbf{U}} = p_S \mathbf{U}, \tag{343}$$

all constraint equations will be satisfied, non-linear terms in equations (338) will vanish, and we will obtain the equation of harmonic oscillator for determination of vector $\mathbf{U}$:

$$\mathbf{U}^{\bullet\bullet} + 4\pi\, \mathbf{U} = 0. \tag{344}$$

From equation (344) follows that vector $\mathbf{U}$ executes harmonic oscillations in the intrinsic system with frequency $\omega_S = \sqrt{4\pi}$. So, Lorentz-invariant pseudo-Euclidean square of the wave vector is equal to 4π ($k^\nu k_\nu = \omega_S^2 = 4\pi$).
The singlet-triplet wave lethos of the form (343), (344) is the three-current analogue of the considered in the article [1] one-current singlet object – "heavy photon".
2. We may take into account that three-current wave (338), similarly to other triplet and singlet-triplet current waves, in contrast to free (zero-current) Yang-Mills waves, does not have a large-scale invariance in the wave amplitude (or energy) – it is possible to distinguish the behavior of small – and large amplitude waves. For small-amplitude waves there is appropriate a procedure of problem linearization, with which in wave equations (338) the non-linear terms of the form $\overset{ab}{\mathrm{I}}\, \overset{b}{\mathbf{U}}$ are omitted as small quantities of the third amplitude order (free Yang-Mills waves are *essentially-nonlinear* and do not allow the

linearization procedure).

In such linearized formulation of the problem we may not subject the variables of the problem to rigid condition (343), and, in this way, remove the strong geometric degeneration of wave (condition (343) transforms the three-dimensional object lethos into an uninteresting one-dimensional structure).

All vectors $\mathbf{U}$ and $\overset{a}{\mathbf{U}}$ of the problem in such approximate linearized formulation execute synchronous harmonic oscillations with frequency $\omega_S = \sqrt{4\pi}$. Vectors $\mathbf{U}$ and $\overset{3}{\mathbf{U}}$ are orthogonal to the plane stretched on vectors $\overset{1}{\mathbf{U}}$ and $\overset{2}{\mathbf{U}}$. Relation $2\ \overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}}= p_S^2\, \mathrm{U}^2$ links the amplitudes of synchronously oscillating motions of vectors $\mathbf{U}$, $\overset{1}{\mathbf{U}}$ and $\overset{2}{\mathbf{U}}$.

In this approximate solution, in contrast to the exact, but geometrically degenerate solution (343), chiral determinant CD $\left(\overset{1}{\mathbf{U}}, \overset{2}{\mathbf{U}}, \overset{3}{\mathbf{U}}\right)$ is not identically zero, and we can submit the solution to the condition of chiral determinacy .

System of equations (338) describes the left-hand three-current wave with time-like wave vector . It is possible to consider a similar wave with space-like wave vector. The solution of such problem (similar to the other wave problems with a space-like vector, considered above) in the intrinsic wave system is some ”frozen transverse ripple” of currents and potentials with unrestricted amplitude growth along the longitudinal wave coordinate. The solution growth can be restricted by imposing the requirement of chiral determinacy of the solution and the condition of restriction of pseudo-Euclidean module of currents.

10.5 Mixed Singlet-triplet Four-current One-neutrino State (the Left-hand Four-current State)

10.5.1 Left-hand Four-current State Lagrangian

Let us consider such a singlet-triplet state, in which one couple of orthogonal currents – left-hand current l^ν and neutrino current N^ν, form both a singlet current J^ν and the third component of YM-triplet of currents $\overset{3}{\mathrm{J}}{}^\nu$:

$$\mathrm{J}^\nu = l^\nu + N^\nu;\ \overset{3}{\mathrm{J}}{}^\nu = l^\nu - N^\nu;\ l^\nu N_\nu = 0;\ N^\nu N_\nu = 0;\ l^\nu l_\nu < 0. \tag{345}$$

Besides this couple of currents in the state under consideration, there must also be available two other YM-currents $\overset{1}{\mathrm{J}}{}^\nu$ and $\overset{2}{\mathrm{J}}{}^\nu$, since their scalar product is nonzero:

$$2\ \overset{1}{\mathrm{J}}{}^\nu\ \overset{2}{\mathrm{J}}_\nu = l^\nu l_\nu. \tag{346}$$

Substitution of relations (345) into the base Lagrangian of the ST-theory (17) results in the following Lagrangian expression of the left-hand four-current problem:

$$L = -\frac{1}{2}m^\nu m_\nu - \frac{1}{2}\mathbf{j}^\nu \mathbf{j}_\nu - m^\nu Q_\nu - n^\nu Z_\nu - \mathbf{j}^\nu \cdot \mathbf{W}_\nu - \frac{1}{16\pi}\left(\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} + \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu}\right) - \frac{1}{2}\lambda\, n^\nu\, n_\nu. \tag{347}$$

In the Lagrangian notation (347) there have been used the same rescaled currents m^ν and $\mathbf{j}^\nu$ (324), (325), which were used in the left-hand three-current Lagrangian notation (326). The linear combinations of singlet and triplet potential Q_ν (322) and Z^ν (258), introduced above, are also used here. Current n^ν in the Lagrangian (347) is a rescaled neutrino current N^ν:

$$n^\nu = \frac{1}{2\,p_T}\,N^\nu. \tag{348}$$

The last term in (347) is a "penalty" for neutrino character of current n^ν; λ is the Lagranges multiplier of this penalty.
The algebraic constraints, imposed on currents (345), (346) in these notations take the form:

$$n^\nu\,n_\nu = 0;\; m^\nu\,n_\nu = 0;\; 2\,\overset{1}{\mathrm{j}}{}^\nu\,\overset{2}{\mathrm{j}}_\nu = p_S^2 m^\nu m_\nu, \tag{349}$$

$$m^\nu\,m_\nu < 0;\; \overset{1}{\mathrm{j}}{}^\nu\,\overset{1}{\mathrm{j}}_\nu < 0;\; \overset{2}{\mathrm{j}}{}^\nu\,\overset{2}{\mathrm{j}}_\nu < 0. \tag{350}$$

From three constraints (349) in the Lagrangian (347) only the condition of current n^ν isotropy is included as a "penalty" term. The Lagrangian extension (347) of the term of the form $\mu\,m^\nu\,n_\nu$ ("penalty" for condition of orthogonality of currents m^ν and n^ν, μ is the Lagrange's multiplier) results in such current equations, with which the conditions $m^\nu\,n_\nu = 0$ and $m^\nu\,m_\nu < 0$ turn out to be incompatible if $\mu \neq 0$.

10.5.2 Field Equations of the Left-hand Four-current State

From the Lagrangian of the form (347) , the following field equations describing the left-hand four-current state, follow:

$$\begin{aligned} &m^\nu + Q^\nu = 0;\\ &\lambda\,n^\nu + Z^\nu = 0; \end{aligned}$$

$$\overset{a}{\mathrm{j}}{}^\nu + \overset{a}{\mathrm{W}}{}^\nu = 0;\; a = 1,\,2. \quad (\text{"current equations"}) \tag{351}$$

$$\partial_\mu \mathrm{W}^{\mu\nu} = 4\pi p_T\,(m^\nu + n^\nu)\,. \quad (\text{"Maxwell equations"}) \tag{352}$$

$$\partial_\mu \mathbf{W}^{\mu\nu} + p_T\,\mathbf{W}_\mu \times \mathbf{W}^{\mu\nu} = 4\pi\left(\mathbf{j}^\nu + p_S\,(m^\nu - n^\nu)\,\overset{3}{\mathbf{e}}\right). \quad (\text{"Yang-Mills equations"}) \tag{353}$$

By extracting the currents from Maxwell and Yang-Mills equations (351), and by extracting auxiliary potentials Q^ν and Z^ν, the problem of the left-hand four-current state can be reduced to the system of wave equations relative to potentials W^ν and $\mathbf{W}^\nu$:

$$\begin{aligned} &-\Box \mathrm{W}^\nu + 4\pi p_T^2\,(1-\mathrm{q})\,\mathrm{W}^\nu + 4\pi\,p_S p_T\,(1+\mathrm{q})\,\overset{3}{\mathrm{W}}{}^\nu = 0;\\ &-\Box\,\overset{a}{\mathrm{W}}{}^\nu + 4\pi\,\overset{a}{\mathrm{W}}{}^\nu + \overset{a}{\mathrm{h}}{}^\nu + \overset{ab}{\mathrm{I}}\,\overset{b}{\mathrm{W}}{}^\nu = 0,\; a = 1,\,2;\\ &-\Box\,\overset{3}{\mathrm{W}}{}^\nu + 4\pi p_S^2\,(1-\mathrm{q})\,\overset{3}{\mathrm{W}}{}^\nu + 4\pi p_S p_T\,(1+\mathrm{q})\,\mathrm{W}^\nu + \overset{3}{\mathrm{h}}{}^\nu + p_T^2\,\overset{3b}{\mathrm{I}}\,\overset{b}{\mathrm{W}}{}^\nu = 0, \end{aligned} \tag{354}$$

where $\mathrm{q} = -1/\lambda$, vector $\mathbf{h}^{\nu}$ is determined in (266), $\overset{ab}{\mathrm{I}}$ is Yang-Mills inertia tensor. Potentials W^{ν} and $\mathbf{W}^{\nu}$ have zero 4-divergences and obey the holonomic constraints following from the conditions (349):

$$\begin{aligned} &p_T^2 \mathrm{W}^{\nu}\mathrm{W}_{\nu} = p_S^2 \overset{3}{\mathrm{W}}{}^{\nu} \overset{3}{\mathrm{W}}_{\nu};\\ &p_S \overset{3}{\mathrm{W}}{}^{\nu} \overset{3}{\mathrm{W}}_{\nu} = p_T \mathrm{W}^{\nu} \overset{3}{\mathrm{W}}_{\nu};\\ &p_T \mathrm{W}^{\nu}\mathrm{W}_{\nu} = p_S \mathrm{W}^{\nu} \overset{3}{\mathrm{W}}_{\nu};\\ &2 \overset{1}{\mathrm{W}}{}^{\nu} \overset{2}{\mathrm{W}}_{\nu} = 4p_T^2 p_S^2 \mathrm{W}^{\nu}\mathrm{W}_{\nu}. \end{aligned} \tag{355}$$

It is obvious that equations (355) are not independent.
Besides the constraints (355), the potentials obey two auxiliary constraints following from the condition for YM-currents (19):

$$\overset{a}{\mathrm{W}}{}^{\nu}\mathrm{R}_{\nu} = 0;\ a = 1,\, 2, \tag{356}$$

where

$$\mathrm{R}_{\nu} = p_S p_T \left(1 + \mathrm{q}\right) \mathrm{W}_{\nu} - \left(p_T^2 + \mathrm{q} p_S^2\right) \overset{3}{\mathrm{W}}_{\nu}\ . \tag{357}$$

10.5.3 Left-hand Four-current State as a Plane Wave

Having an intention to construct the solution to the system of wave equations (354) in the form of a plane wave, we have to consider only transverse waves with space-like vector k^{ν}. (The presence of isotropic vectors n^{ν} and Z^{ν} in the problem excepts the existence of a wave with a time-like wave vector). In the plane wave, vector $\overset{a}{\mathrm{h}}{}^{\nu}$ vanishes. In the wave intrinsic system, D'Alembert operator $\Box$ is reduced to the second longitudinal coordinate derivative. By using complex three-dimensional YM-vectors $\boldsymbol{\tau}$, $\overset{a}{\boldsymbol{\tau}}$ instead of 4-potentials W^{ν} and $\overset{a}{\mathrm{W}}{}^{\nu}$, the system of equations (354) can be reduced to the system of ordinary differential equations:

$$\begin{aligned} &-\boldsymbol{\tau}'' + 4\pi p_T^2 \left(1 - \mathrm{q}\right) \boldsymbol{\tau} + 4\pi p_S p_T \left(1 + \mathrm{q}\right) \overset{3}{\boldsymbol{\tau}} = 0;\\ &-\overset{a}{\boldsymbol{\tau}}{}'' + 4\pi \overset{a}{\boldsymbol{\tau}} + \overset{b}{\boldsymbol{\tau}} \times \left(\overset{a}{\boldsymbol{\tau}} \times \overset{b}{\boldsymbol{\tau}}\right) = 0;\ a = 1,\, 2\\ &-\overset{3}{\boldsymbol{\tau}}{}'' + 4\pi p_S^2 \left(1 - \mathrm{q}\right) \overset{3}{\boldsymbol{\tau}} + 4\pi p_S p_T \left(1 + \mathrm{q}\right) \boldsymbol{\tau} + \overset{b}{\boldsymbol{\tau}} \times \left(\overset{3}{\boldsymbol{\tau}} \times \overset{b}{\boldsymbol{\tau}}\right) = 0. \end{aligned} \tag{358}$$

Coupling equations (355) and (356) for vectors $\boldsymbol{\tau}$, $\overset{a}{\boldsymbol{\tau}}$ take the form:

$$\begin{aligned} &2p_T^2 \boldsymbol{\tau} \cdot \boldsymbol{\tau} = p_S^2 \overset{3}{\boldsymbol{\tau}} \cdot \overset{3}{\boldsymbol{\tau}}\\ &p_S \overset{3}{\boldsymbol{\tau}} \cdot \overset{3}{\boldsymbol{\tau}} = p_T \boldsymbol{\tau} \cdot \overset{3}{\boldsymbol{\tau}};\\ &p_T \boldsymbol{\tau} \cdot \boldsymbol{\tau} = p_S \boldsymbol{\tau} \cdot \overset{3}{\boldsymbol{\tau}};\\ &\overset{1}{\boldsymbol{\tau}} \cdot \overset{2}{\boldsymbol{\tau}} = 4p_T^2 p_S^2 \boldsymbol{\tau} \cdot \boldsymbol{\tau}, \end{aligned} \tag{359}$$

and

$$\overset{a}{\boldsymbol{\tau}} \cdot \boldsymbol{\rho} = 0;\ a = 1,\ 2, \tag{360}$$

where

$$\boldsymbol{\rho} = p_S p_T \left(1 + \mathrm{q}\right) \boldsymbol{\tau} - \left(p_T^2 + \mathrm{q} p_S^2\right) \overset{3}{\boldsymbol{\tau}}\ . \tag{361}$$

Besides holonomic constraints (359), (360), nonholonomic constraints, which are sufficient to take into account in the initial conditions for (358), are also imposed on the system of motion (358):

$$\overset{abc}{\varepsilon} \overset{b}{\boldsymbol{\tau}} \cdot \overset{c}{\boldsymbol{\tau}}{}' = 0;\ a = 1,\ 2,\ 3. \tag{362}$$

The system of motion (358) is so overloaded with auxiliary algebraic conditions (359), (360), that the very fact of the existence of solutions is doubtful. But if solutions (358), satisfying the constraints (359), (360), do exist, they grow unrestrictedly along the longitudinal coordinate of the wave. To restrict the solution growth, like in the other wave problems, it is possible to use the condition of chiral determinacy of the solution and the condition of restriction of pseudo-Euclidean module of currents.

10.6 Total Four-current Wave State

The base Lagrangian of four-current state is determined by the formula (17):

$$L = -\frac{1}{8p_S^2}\mathrm{J}^\nu \mathrm{J}_\nu - \frac{1}{8p_T^2}\mathbf{J}^\nu \cdot \mathbf{J}_\nu - \frac{1}{2p_S}\mathrm{J}^\nu \mathrm{W}_\nu - \frac{1}{2p_T}\mathbf{J}^\nu \cdot \mathbf{W}_\nu - \frac{1}{16\pi}\left(\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} + \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu}\right). \tag{363}$$

Additional algebraic conditions, imposed on the four-current state, are determined by formulas (20) and (32):

$$\begin{aligned} &\overset{3}{\mathrm{J}}{}^\nu \left(\overset{3}{\mathrm{J}}_\nu - \mathrm{J}_\nu\right) = 0; \\ &2\, \overset{1}{\mathrm{J}}{}^\nu\, \overset{2}{\mathrm{J}}_\nu = \overset{3}{\mathrm{J}}{}^\nu\, \overset{3}{\mathrm{J}}_\nu\ . \end{aligned} \tag{364}$$

Like in other wave problems, considered above, at the derivation of the field equations, we will directly use the base Lagrangian (363), without "extending" it with additional conditions up to the *"effective"* Lagrangian (36). By doing so, we interpret the conditions (364) as *"natural constraints"* for some of the four-current states and as the conditions for forbidding the existence of other states which are incompatible with the constraints (364).

The Lagrangian (363) generates the following current and field equations of the four-current state:

$$\begin{aligned} &\mathrm{J}^\nu + 2p_S \mathrm{W}^\nu = 0; \\ &\mathbf{J}^\nu + 2p_T \mathbf{W}^\nu = 0; \\ &(\text{"current equations"}). \end{aligned} \tag{365}$$

$$\partial_\mu \mathrm{W}^{\mu\nu} = \frac{2\pi}{p_S}\mathrm{J}^\nu, \quad (\text{"Maxwell equations"}) \tag{366}$$

$$\partial_\mu \mathbf{W}^{\mu\nu} + p_T\, \mathbf{W}_\mu \times \mathbf{W}^{\mu\nu} = \frac{2\pi}{p_T}\mathbf{J}^\nu . \text{(”Yang-Mills equations”)} \tag{367}$$

By extracting currents from field equations (366) and (367), by means of current equations (365), it is possible to reduce the problem of four-current state to the system of wave equations relative to the potentials W^ν and $\mathbf{W}^\nu$:

$$-\Box \mathrm{W}^\nu + 4\pi \mathrm{W}^\nu = 0; \tag{368}$$

$$-\Box \mathbf{W}^\nu + 4\pi \mathbf{W}^\nu + p_T \mathbf{h}^\nu + p_T^2 \hat{\mathrm{I}} \cdot \mathbf{W}^\nu = 0, \tag{369}$$

where, as previously,

$$\mathbf{h}^\nu = \mathbf{W}_\mu \times (2\partial^\mu \mathbf{W}^\nu - \partial^\nu \mathbf{W}^\mu)\, ;$$

$$\overset{ab}{\mathrm{I}} = \overset{a}{\mathrm{W}}{}^\mu \overset{b}{\mathrm{W}}_\mu - \overset{ab}{\delta} \overset{c}{\mathrm{W}}_\mu \overset{c}{\mathrm{W}}{}^\mu .$$

The solutions to wave equations (368) and (369) obey the conditions of zero 4-divergence of potentials:

$$\partial_\mu \mathrm{W}^\mu = 0;\ \partial_\mu \mathbf{W}^\mu = 0.$$

Wave equations (368) and (369) are independent from each other, but their solutions obey the constraints following from (364) and (365):

$$p_T \overset{3}{\mathrm{W}}{}^\nu \overset{3}{\mathrm{W}}_\nu = p_S \overset{3}{\mathrm{W}}{}^\nu \mathrm{W}_\nu ; \tag{370}$$

$$2 \overset{1}{\mathrm{W}}{}^\nu \overset{2}{\mathrm{W}}_\nu = \overset{3}{\mathrm{W}}{}^\nu \overset{3}{\mathrm{W}}_\nu \ . \tag{371}$$

By constructing solution to the four-current problem in the form of a plane wave with time-like wave vector, we can reduce equations (368) and (369) to the system of ordinary differential equations relative to four three-dimensional vectors $\mathbf{U}$, $\overset{a}{\mathbf{U}}$ in the wave frame of reference (the dot next to the letter stands for a time derivative in the wave intrinsic system):

$$\mathbf{U}^{\bullet\bullet} + 4\pi \mathbf{U} = 0; \tag{372}$$

$$\overset{a}{\mathbf{U}}{}^{\bullet\bullet} + 4\pi \overset{a}{\mathbf{U}} + p_T^2 \overset{ab}{\mathrm{I}} \overset{b}{\mathbf{U}} = 0,\ (a = 1,\, 2,\, 3)\, . \tag{373}$$

In the wave intrinsic system there are no time components of potentials:

$$\mathrm{W}^\nu = \{0;\, \mathbf{U}\}\, ;\ \overset{a}{\mathrm{W}}{}^\nu = \left\{0;\, \overset{a}{\mathbf{U}}\right\} .$$

Yang-Mills inertia tensor has the form:

$$\overset{ab}{\mathrm{I}} = \overset{ab}{\delta} \overset{c}{\mathbf{U}} \cdot \overset{c}{\mathbf{U}} - \overset{a}{\mathbf{U}} \cdot \overset{b}{\mathbf{U}} \ .$$

Coupling equations ((370), (371) take the form:

$$p_T \overset{3}{\mathbf{U}}{}^2 = p_S \overset{3}{\mathbf{U}} \cdot \mathbf{U}; \tag{374}$$

$$2 \overset{1}{\mathbf{U}} \cdot \overset{2}{\mathbf{U}} = \overset{3}{\mathbf{U}}{}^2. \tag{375}$$

The system of equations (372), (373) with conditions (374), (375) has one obvious and rather primitive solution of the following form:

$$\overset{1}{\mathbf{U}} = \frac{1}{\sqrt{2}}\mathbf{V};\ \overset{2}{\mathbf{U}} = \frac{1}{\sqrt{2}}\mathbf{V};\ \overset{3}{\mathbf{U}} = \mathbf{V};\ \mathbf{U} = \frac{p_T}{p_S}\mathbf{V}. \tag{376}$$

For solution (376) all four vectors $\mathbf{U}$, $\overset{a}{\mathbf{U}}$ are parallel, the coupling equations (374) and (375) are satisfied, non-linear terms in (377) vanish:

$$\overset{ab}{\mathrm{I}}\,\overset{b}{\mathbf{U}} = 0, \tag{377}$$

vector $\mathbf{V}$ is a harmonic vector, oscillating at frequency $\sqrt{4\pi}$:

$$\mathbf{V}^{\bullet\bullet} + 4\pi\mathbf{V} = 0. \tag{378}$$

Four-current wave (376), (378) disappoints with its primitiveness it is just a four-time replicated "heavy photon" considered in the article [1]. In this wave, Yang-Mills' non-linearity vanishes due to relation (377). But the constraint condition, coordinating the dynamics of a nonlinear triplet oscillator (374) with the dynamics of a linear singlet oscillator (374), is so rigid that, apparently, there are no other four current states besides the state (376), (378): the four currents of the singlet-triplet theory are too "tight" in the same point of the four-dimensional space-time continuum. This fact causes trouble and doubt of the usefulness of the four-current Lagrangian itself.

If we afford to neglect the condition (374), intermixing currents of the singlet and triplet sectors, there appears possibility in the triplet sector to construct the waves of a more complicated form than a linear wave (376). For example, it is possible to construct a solution, where vector $\overset{3}{\mathbf{U}}$ is orthogonal to vectors $\overset{1}{\mathbf{U}}$ and $\overset{2}{\mathbf{U}}$ which coincide with each other:

$$\overset{1}{\mathbf{U}} = \overset{2}{\mathbf{U}} = u\,\{1;\,0;\,0\};\ \overset{3}{\mathbf{U}} = u\,\{0;\,0;\,\sqrt{2}\}. \tag{379}$$

For the wave of the form (379) the condition of triplet auto-normalization (375) is satisfied, and Yang-Mills inertia tensor has the form:

$$\hat{\mathrm{I}} = u^2 \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \tag{380}$$

Substitution of (379) and (380) into Yang-Mills equations (373) gives Duffing equation for the amplitude function u:

$$u^{\bullet\bullet} + 4\pi u + p_T^2 u^3 = 0. \tag{381}$$

The system described by (379), (381) can be named Duffing-Yang-Mills' oscillator (duymos). The object duymos, undoubtedly, belongs to Yang-Mills' *mathematics*, however it does not exist as an object of Maxwell-Yang-Mills' singlet-triplet *physics*, in view of impossibility to construct a solution to Maxwell's equations (372) which would match it by (374).

Summary

In the present article we have made an extensive attempt of a consistent interpretation of the singlet-triplet theory as a sector of the *classical* field theory. Each field (both singlet and triplet) is treated as a *dyad* consisting of a current and a potential. Particles and waves emerge as solutions to field equations. There have been considered a number of wave states, – in contrast to electrodynamics, there are about two dozens of wave types in the singlet-triplet theory. Numerical investigations of some wave states will be provided in the other articles of this series. It is unknown how to compare these objects of the theory with some objects of experimental physics that allow observation.

The most vulnerable point of the theory, developed here, is the absence of theorems of the existence of stationary problems solutions. It is these problems that provide a classical description of massive elementary particles. If stationary states exist, the corresponding solutions can be constructed numerically. If solutions to stationary problems do not exist, the current part of the Lagrangian has to be complicated with addition of, for example, an expression, quadratic in the tensor of currents $\mathrm{G}_{\mu\nu}$, $\mathbf{G}_{\mu\nu}$, into the Lagrangian of each sector of physics:

$$\mathrm{G}_{\mu\nu} = \partial_\mu \mathrm{J}_\nu - \partial_\nu \mathrm{J}_\mu, \text{(singlet current tensor)}$$
$$\mathbf{G}_{\mu\nu} = \partial_\mu \mathbf{J}_\nu - \partial_\nu \mathbf{J}_\mu + p_T \mathbf{J}_\mu \times \mathbf{J}_\nu \text{ (triplet current tensor).}$$

However, this Lagrangian extension dramatically changes the current equations of the theory, and to some extent makes the postulate of space-like character of any currents that appear in the classical field theory, less convincing.

Hopefully, among the readers of this article there will appear a mathematician able to formulate correctly and prove the existence theorem for stationary states.

The second vulnerable point of the theory is the method of accounting the chirality problem suggested in the article (soft chiralization procedure). This method forms unsmooth solutions with derivative discontinuities. Not every physicist will agree to pay such a price for the chiral definitiveness of solutions to field equations.

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