7

Taguchi's Quality Engineering

Introduction

Most of the body of knowledge associated with the quality sciences was developed in the United Kingdom as design of experiments and in the United States as statistical quality control. More recently, Dr. Genichi Taguchi has added to this body of knowledge. In particular, he introduced the loss function concept, which combines cost, target, and variation into one metric with specifications being of secondary importance. Furthermore, he developed the concept of robustness, which means that noise factors are taken into account to ensure that the system functions correctly. Noise factors are uncontrollable variables that can cause significant variability in the process or the product. Taguchi is a mechanical engineer and has won four Deming Awards.

Loss Function

Taguchi has defined quality as the loss imparted to society from the time a product is shipped. Societal losses include failure to meet customer requirements, failure to meet ideal performance, and harmful side effects. Many practitioners have included the losses due to production, such as raw material, energy, and labor consumed on unusable products or toxic by-products.

The loss-to-society concept can be illustrated by an example associated with the production of large vinyl covers to protect materials from the elements. Figure 7.1 shows three stages in the evolution of vinyl thickness. At (1), the process is just capable of

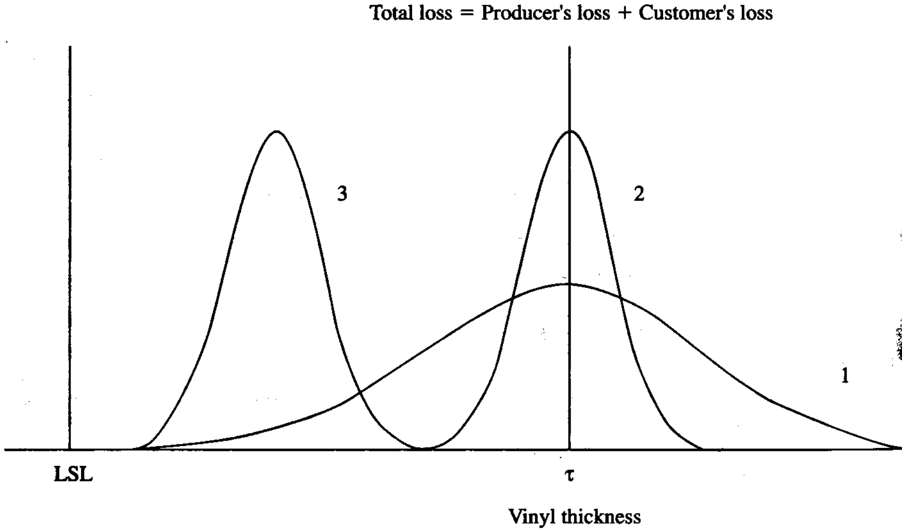


Figure 7.1 Loss to Society
Reproduced, with permission, from *Taguchi Methods: Introduction to Quality Engineering* (Allen Park, Mich.: American Supplier Institute, Inc., 1991).

meeting the specifications (USL and LSL); however, it is on the target *tau,* τ.[1] After considerable effort, the production process was improved by reducing the variability about the target, as shown at (2). In an effort to reduce its production costs, the organization decided to shift the target closer to the LSL, as shown at (3). This action did result in a substantial improvement by lowering the cost to the organization; however, the vinyl covers were not as strong as before. When farmers used the covers to protect wheat from the elements, they tore and a substantial loss occurred to the farmers. In addition, the cost of wheat increased as a result of supply-and-demand factors, thereby causing an increase in wheat prices and a further loss to society. The company's reputation suffered, which created a loss of market share with its unfavorable loss aspects.

Assuming the target is correct, losses of concern are those caused by a product's critical performance characteristics deviating from the target. The importance of concentrating on "hitting the target" is documented by Sony. In spite of the fact that the design and specifications were identical, U.S. customers preferred the color density of shipped TV sets produced by Sony—Japan over those produced by Sony—USA. Investigation of this situation revealed that the frequency distributions were markedly different, as shown in Figure 7.2 . Even though Sony—Japan had 0.3% outside the specifications,

[1] Taguchi uses the symbol *m* for the target.

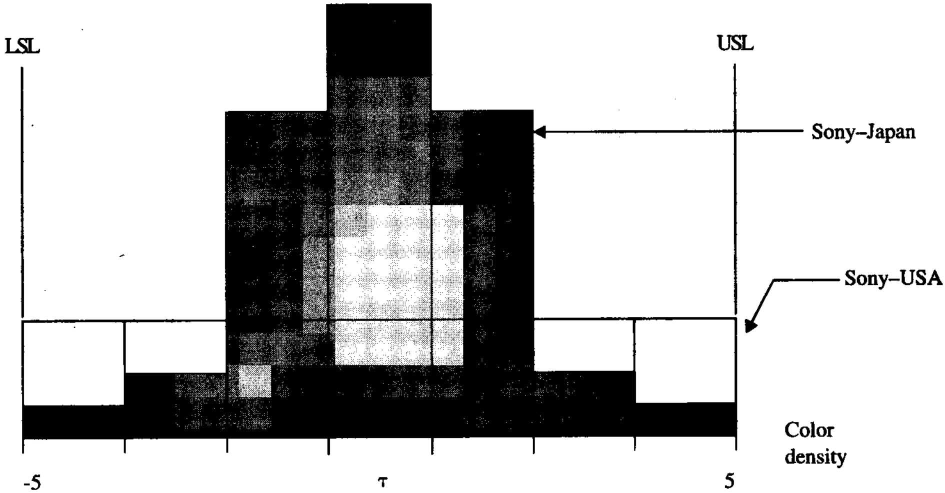


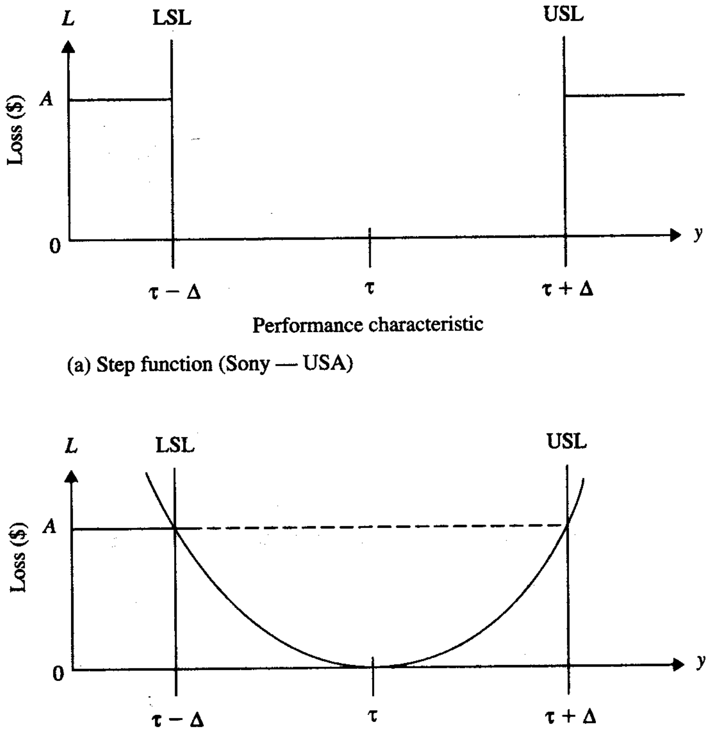
Figure 7.2 Distribution of Color Density for Sony- USA and Sony - Japan
Source: *The Asahi,* April 17, 1979.

the distribution was normal and centered on the target. The distribution of the Sony—USA was uniform between the specifications with no values outside specifications. It was clear that customers perceived quality as meeting the target (Japan) rather than just meeting the specifications (USA). Ford Motor Company had a similar experience with transmissions.

Out of specification is the common measure of quality loss. Although this concept may be appropriate for accounting, it is a poor concept for all other areas. It implies that all products that meet specifications are good, whereas those that do not are bad. From the customer's point of view, the product that barely meets specification is as good (or bad) as the product that is barely out of specification. It appears we are using the wrong measuring system. The loss function corrects for the deficiency described above by combining cost, target, and variation into one metric.

Nominal-the-Best

Although Taguchi has developed more than 68 loss functions, many situations are approximated by the quadratic function which is called the nominal-the-best type. Figure 7.3 (a) shows the step function that describes the Sony—USA situation. When the value for the performance characteristic, y, is within specifications the loss is \$0, and when it is outside the specifications the loss is \$$A$. The quadratic function is shown at 7.3 (b) and describes the Sony—Japan situation. In this situation loss occurs as soon as the performance characteristic, y, departs from the target, τ.



(a) Step function (Sony — USA)

(b) Quadratic function (Sony — Japan)

Figure 7.3 Step and Quadratic Loss Functions

The quadratic loss function is described by the equation

$$L = k(y - \tau)^2$$

where L = cost incurred as quality deviates from the target

y = performance characteristic

τ = target

k = quality loss coefficient

The loss coefficient is determined by setting $\Delta = (y - \tau)$, the deviation from the target. When Δ is at the USL or LSL, the loss to the customer of repairing or discarding the product is \$$A$. Thus,

$$k = A/(y - \tau)^2 = A/\Delta^2$$

EXAMPLE PROBLEM

If the specifications are 10 ± 3 for a particular quality characteristic and the average repair cost is $230, determine the loss function. Determine the loss at $y = 12$.

$$k = A/\Delta^2 = 230/3^2 = 25.6$$

Thus, $L = 25.6\,(y - 10)^2$ and at $y = 12$,

$$\begin{aligned} L &= 25.6(y - 10)^2 \\ &= 25.6(12 - 10)^2 \\ &= \$102.40 \end{aligned}$$

Average Loss

The loss described here assumes that the quality characteristic is static. In reality, you can't always hit the target, τ. It is varying due to noise, and the loss function must reflect the variation of many pieces rather than just one piece. Noise factors are classified as external and internal, with internal being further classified as unit-to-unit and deterioration.

A refrigerator temperature control will serve as an example to help clarify the noise concept. External noise is due to the actions of the user, such as the number of times the door is opened and closed, amount of food inside, the initial temperature, etc. Unit-to-unit internal noise is due to variation during production such as seal tightness, control sensor variations, etc. Although this type of noise is inevitable, every effort should be made to keep it to a minimum. Noise due to deterioration is caused by leakage of refrigerant, mechanical wear of compressor parts, etc. This type of noise is primarily a function of the design. Noise factors cause deviation from the target, which causes a loss to society.

Figure 7.4 shows the nominal-the-best loss function with the distribution of the noise factors. An equation can be derived by summing the individual loss values and dividing by their number to give

$$\bar{L} = k\,[\sigma^2 + (\bar{y} - \tau)^2]$$

where $\bar{L}$ = the average or expected loss.

Because the population standard deviation, σ, will rarely be known, the sample standard deviation, s, will need to be substituted. This action will make the value somewhat larger; however, the average loss is a very conservative value.

The loss can be lowered by first reducing the variation, σ, and then adjusting the average, $\bar{y}$, to bring it on target, τ. The loss function speaks the language of things,

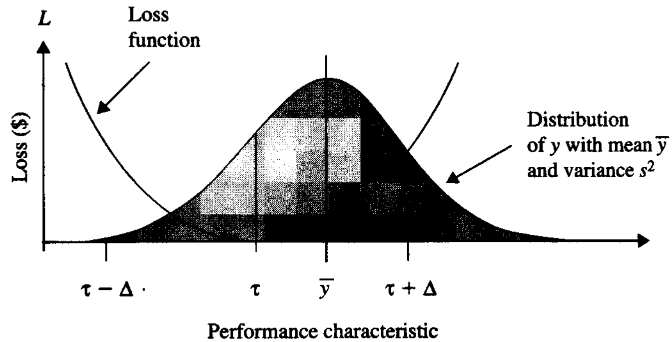


Figura 7.4 Average or Expected Loss

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which is engineering's measure, and money, which is management's measure. Examples where the nominal-the-best loss function would be applicable are the performance characteristics of color density, voltage, dimensions, etc.

EXAMPLE PROBLEM

Compute the average loss for a process that produces steel shafts. The target value is 6.40 mm and the coefficient is 9500. Eight samples give 6.36, 6.40, 6.38, 6.39, 6.43, 6.39, 6.46, and 6.42.

$$s = 0.0315945 \qquad \bar{y} = 6.40375$$

$$\begin{aligned}\overline{L} &= k\left[s^2 + (\bar{y} - \tau)^2\right] \\ &= 9500\left[0.0315945^2 + (6.40375 - 6.40)^2\right] \\ &= \$9.62\end{aligned}$$

Other Loss Functions

There are two other loss functions that are quite common, smaller-the-better and larger-the-better. Figure 7.5 illustrates the concepts.

As can be seen by the figure, the target value for smaller-the-better is 0 and there are no negative values for the performance characteristic. Examples of performance char-

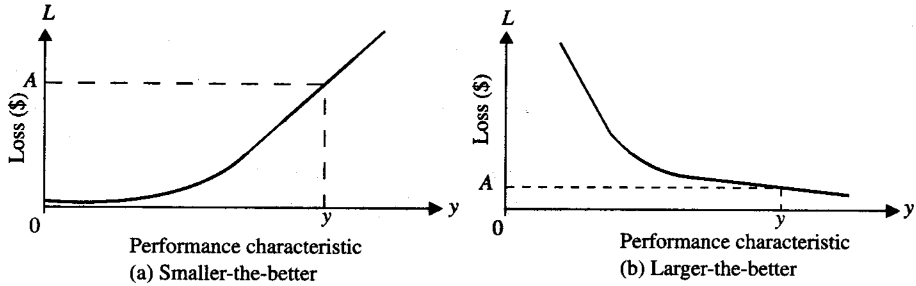

Figure 7.5 Smaller-the-Better and Larger-the-Better Loss Functions

TABLA 7.1
Summary of the Equations for the Three Common Loss Functions

Nominal-the-best	$L = k\,(y - \tau)^2$ where $k = A/\Delta^2$
	$\bar{L} = k\,(\text{MSD})$ where $\text{MSD} = [\Sigma\,(y - \tau)^2]/n$
	$\bar{L} = k[\sigma^2 + (\bar{y} - \tau)^2]$
Smaller-the-better	$L = ky^2$ where $k = A/y^2$
	$\bar{L} = k\,(\text{MSD})$ where $\text{MSD} = [\Sigma y^2]/n$
	$\bar{L} = k[\bar{y}^2 + \sigma^2]$
Larger-the-better	$L = k(1/y^2)$ where $k = Ay^2$
	$\bar{L} = k\,(\text{MSD})$ where $\text{MSD} = [\Sigma(1/y^2)]/n$
	$\bar{L} = k[\Sigma(1/y^2)]/n$

acteristics are radiation leakage from a microwave appliance, response time for a computer, pollution from an automobile, out of round for a hole, etc.

In the larger-the-better situation, shown in Figure 7.5 (b), the target value is ∞, which gives a zero loss. There are no negative values and the worst case is at $y = 0$. Actually, larger-the-better is the reciprocal of smaller-the-better, and because of the difficulty of working with ∞, some practitioners prefer to work with the reciprocal. Thus, a larger-the-better performance characteristic of meters/second becomes a smaller-the-better performance characteristic of seconds/meter. Examples of performance characteristics are bond strength of adhesives, welding strength, automobile gasoline consumption, etc.

Summary of the Equations

Table 7.1 gives a summary of the equations for the three common loss functions. It also shows the relationship of the loss function to the mean squared deviation (MSD).

These three common loss functions will cover most situations. After selecting one of the loss functions, one point on the curve needs to be determined in order to obtain the coefficient. It is helpful to work with accounting to obtain this one point. Knowing the coefficient, the equation is complete and can be used to justify the use of resources and as a benchmark to measure improvement. It is much easier to use the loss function to obtain cost information than to develop an elaborate quality cost system. Cost data are usually quite conservative; therefore, it is not necessary for the loss function to be perfect for it to be effective.

Sometimes the loss function curves are modified for particular situations. For example, larger-the-better can be represented by one-half the nominal-the-best curve. Another situation occurs where the performance characteristic is weld strength. In such a case the larger-the-better curve can terminate at the strength of the parent metal rather than ∞. If the three common loss functions do not seem to be representative of a particular situation, then individual points can be plotted.

Orthogonal Arrays[2]

Orthogonal arrays (OA) are a simplified method of putting together an experiment. The original development of the concept was by Sir R. A. Fischer of England in the 1930s. Taguchi added three OAs to the list in 1956, and the National Institute of Science and Technology (NIST) of the United States added three.

An orthogonal array is shown in Table 7.2 . The 8 in the designation OA8 represents the number of rows, which is also the number of treatment conditions (TC) and the degrees of freedom. Across the top of the orthogonal array is the maximum number of factors that can be used, which in this case is seven. The levels are designated by 1 and 2. If more levels occur in the array, then 3, 4, 5, etc., are used. Other schemes such as $-$, 0, and $+$ can be used.

The orthogonal property of the OA is not compromised by changing the rows or the columns. Taguchi changed the rows from a traditional design so that TC 1 was composed of all level 1s and, if the team desired, could thereby represent the existing conditions. Also, the columns were switched so that the least amount of change occurs in the columns on the left. This arrangement can provide the team with the capability to assign factors with long setup times to those columns. Orthogonal arrays can handle dummy factors and can be modified. The reader is referred to the bibliography for these techniques.

[2] Orthogonal arrays, interaction tables, and linear graphs in this chapter are reproduced, with permission, from *Taguchi Methods: Introduction to Quality Engineering* (Allen Park, Mich.: American Supplier Institute, Inc., 1991).

TABLE 7.2

Orthogonal Array (OA8)*

TC	*1*	*2*	*3*	*4*	*5*	*6*	*7*
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

*Taguchi uses a more elaborate system of identification for the orthogonal arrays. It is the authors' opinion that a simple system using OA is more than satisfactory.

To determine the appropriate orthogonal array, use the following procedure:

1. Define the number of factors and their levels.
2. Determine the degrees of freedom.
3. Select an orthogonal array.
4. Consider any interactions.

The first step is completed by the project team.

Degrees of Freedom

The number of degrees of freedom is a very important value because it determines the minimum number of treatment conditions. It is equal to the sum of

(Number of levels − 1) for each factor

(Number of levels − 1)(number of levels − 1) for each interaction

One for the average

An example problem will illustrate the concept.

EXAMPLE PROBLEM

Given four two-level factors, *A, B, C, D,* and two suspected interactions, *BC* and *CD*, determine the degrees of freedom, df. What is the answer if the factors are three-level?

$$\text{df} = 4(2 - 1) + 2(2 - 1)(2 - 1) + 1 = 7$$
$$\text{df} = 4(3 - 1) + 2(3 - 1)(3 - 1) + 1 = 17$$

At least seven treatment conditions are needed for the two-level, and 17 for the three-level. As can be seen by the example, the number of levels has considerable influence on the number of treatment conditions. Although a three-level design provides a great deal more information about the process, it can be costly in terms of the number of treatment conditions.

The maximum degrees of freedom is equal to

$$\text{df} = l^f$$

where l = number of levels
f = number of factors

For the example problem with two levels, df $= 2^4 = 16$. Table 7.3 shows the maximum degrees of freedom.

In the example problem, it was assumed that four of the two-factor interactions (*AB, AC, AD,* and *BD*), four of the three-factor interactions (*ABC, ABD, ACD,* and *BCD*), and the four-factor interaction (*ABCD*) would not occur. Interactions are discussed later in the chapter.

TABLE 7.3

Maximum Degrees of Freedom for a Four-Factor, Two-Level Experimental Design

Design Space				*df*
A	*B*	*C*	*D*	4
AB	*AC*	*AD*	*BC*	6
BD	*CD*			
ABC	*ABD*	*ACD*	*BCD*	4
ABCD				1
Average				1
			Sum	16

Selecting the Orthogonal Array

Once the degrees of freedom are known, the next step, selecting the orthogonal array (OA), is quite easy. The number of treatment conditions is equal to the number of rows in the OA and must be equal to or greater than the degrees of freedom. Table 7.4 shows the orthogonal arrays that are available, up to OA36. Thus, if the number of degrees of freedom is 13, then the next available OA is OA16. The second column of the table has the number of rows and is redundant with the designation in the first column. The third column gives the maximum number of factors that can be used, and the last four columns give the maximum number of columns available at each level.

Analysis of the table shows that there is a geometric progression for the two-level arrays of OA4, OA8, OA16, OA32, . . . , which is 2^2, 2^3, 2^4, 2^5, . . . , and for the

TABLE 7.4
Orthogonal Array Information

OA	*Number of Rows*	*Maximum Number of Factors*	MAXIMUM NUMBER OF COLUMNS			
			2-Level	*3-Level*	*4-Level*	*5-Level*
OA2	4	3	3	—	—	—
OA8	8	7	7	—	—	—
OA9	9	4	—	4	—	—
OA12	12	11	11	—	—	—
OA16	16	15	15	—	—	—
OA16′	16	5	—	—	5	—
OA18	18	8	1	7	—	—
OA25	25	6	—	—	—	6
OA27	27	13	—	13	—	—
OA32	32	31	31	—	—	—
OA32′	32	10	1	—	9	—
OA36	36	23	11	12	—	—
OA36′	36	16	3	13	—	—
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.

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three-level arrays of OA9, OA27, OA81, . . . , which is 3^2, 3^3, 3^4, ... Orthogonal arrays can be modified, and the reader is referred to the references for that information.

Interaction Table

Confounding is the inability to distinguish among the effects of one factor from another factor and/or interaction. In order to prevent confounding, we need to know which columns to use for the factors. This knowledge is provided by an interaction table, which is shown in Table 7.6 . The orthogonal array (OA8) is repeated in Table 7.5 for the convenience of the reader.

Let's assume that factor *A* is assigned to column 1 and factor *B* to column 2. If there is an interaction between factors *A* and *B,* then column 3 is used for the interaction, *AB.*

TABLE 7.5
Orthogonal Array OA8

TC	*1*	*2*	*3*	*4*	*5*	*6*	*7*
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

TABLE 7.6
Interaction Table for OA8

Column	*1*	*2*	*3*	*4*	*5*	*6*	*7*
1	(1)	3	2	5	4	7	6
2		(2)	1	6	7	4	5
3			(3)	7	6	5	4
4				(4)	1	2	3
5					(5)	3	2
6						(6)	1
7							(7)

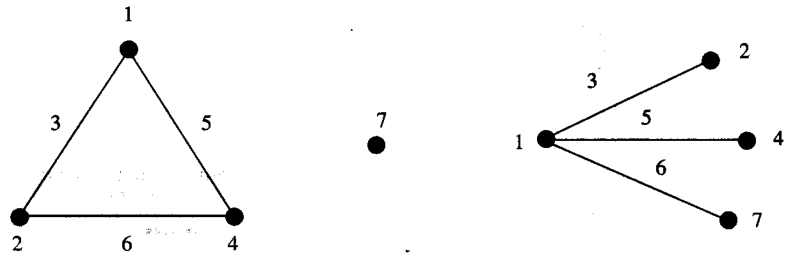


Figure 7.6 Linear Graphs for OA8

Another factor, say, *C*, would need to be assigned to column 4. If there is an interaction between factor *A* (column 1) and factor *C* (column 4), then interaction *AC* will occur in column 5. The columns that are reserved for interactions are used so that calculations can be made to determine whether there is a strong interaction. If there are no interactions, then all the columns can be used for factors. The actual experiment is conducted using the columns designated for the factors, and these columns are referred to as the design matrix. All the columns are referred to as the design space.

Linear Graphs

Taguchi developed a simpler method of working with interactions using linear graphs. Two are shown in Figure 7.6 for OA8. They make it easier to assign factors and interactions to the various columns of an array. Factors are assigned to the points. If there is an interaction between two factors, then it is assigned to the line segment between the two points. For example, using the linear graph on the left in the figure, if factor *B* is assigned to column 2 and factor *C* is assigned to column 4, then interaction *BC* is assigned to column 6. If there is no interaction, then column 6 can be used for a factor.

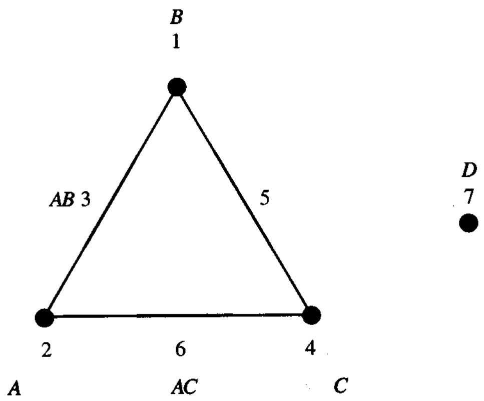
The linear graph on the right would be used when one factor has three two-level interactions. Three-level orthogonal arrays must use two columns for interactions, because one column is for the linear interaction and one column is for the quadratic interaction. The linear graphs—and, for that matter, the interaction tables—are not designed for three or more factor interactions, which rarely occur. Linear graphs can be modified; the reader is referred to the references for modification techniques. Use of the linear graphs requires some trial-and-error activity, and a number of solutions may be possible, as shown by the example problem.

EXAMPLE PROBLEM

An experimental design has four two-level factors (*A*, *B*, *C*, *D*) where only main effects are possible for factor *D* and there is no *BC* interaction. Thus, only interactions *AB* and *AC* are possible, and they can be assigned the line segments 3 and 5, 3 and 6, or 5 and 6, with their apex for factor *A*. Factors *B* and *C* are then assigned to the adjacent points. Column 7 or a line segment that does not have an interaction is used for factor *D*. A number of solutions are possible; one is shown here. The one chosen might well be a function of the setup time when the experiment is run. Column 5 is not used, so it is given the symbol *UX* for unexplained, and calculations for this column should show no effect (very small variation).

Orthogonal Array OA8

	B	*A*	*AB*	*C*	*UX*	*AC*	*D*
TC	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2



Interactions

The fourth step in the procedure is to consider interactions. Figure 7.7 shows the graphical relationship between two factors. At (a) there is no interaction because the lines are parallel; at (b) there is some interaction; and at (c) there is a strong interaction.